THE ONTARIO CURRICULUM

GRADES 9–12

Mathematics

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Educators should be aware that, with the exception of **the Grade 9 mathematics course, 2021 (MTH1W)**, the 2005 Mathematics curriculum for Grade 10 and the 2007 Mathematics curriculum for Grades 11–12 remain in effect. All secondary mathematics courses for Grades 10–12 will continue to be based on those documents. All references to Grade 9 that appear in *The Ontario Curriculum, Grades 9 and 10: Mathematics, 2005* and *The Ontario Curriculum, Grades 11 and 12: Mathematics, 2007* have been superseded by *The Ontario Curriculum, Grade 9: Mathematics, 2021*. Addenda have been issued to the Grade 10 MPM2D and MFM2P courses, to be implemented for the 2022–23 school year.

Contents

MTH1W - Mathematics, Grade 9

De-streamed

Issued: 2021

This course enables students to consolidate, and continue to develop, an understanding of mathematical concepts related to number sense and operations, algebra, measurement, geometry, data, probability, and financial literacy. Students will use mathematical processes, mathematical modelling, and coding to make sense of the mathematics they are learning and to apply their understanding to culturally responsive and relevant real-world situations. Students will continue to enhance their mathematical reasoning skills, including proportional reasoning, spatial reasoning, and algebraic reasoning, as they solve problems and communicate their thinking.
Prerequisite: None

Introduction

Preface

This curriculum policy presents the compulsory Grade 9 mathematics course, 2021 (MTH1W). This course supersedes the two Grade 9 courses outlined in *The Ontario Curriculum, Grades 9 and 10: Mathematics, 2005* as well as *The Ontario Curriculum: Mathematics – Mathematics Transfer Course, Grade 9, Applied to Academic, 2006.* Effective September 2021, all mathematics programs for Grade 9 will be based on the expectations outlined on this site.

The Grade 9 mathematics curriculum focuses on key mathematics concepts and skills, as well as on making connections between related math concepts, between mathematics and other disciplines, and between mathematics and the lived experiences of students. This curriculum is designed to support all students in developing an understanding of, and the ability to apply, the range of mathematical knowledge and skills appropriate for the grade level. Consequently, this curriculum is intended to support all students in continuing to build confidence in approaching mathematics, develop a positive attitude towards mathematics, think critically, work collaboratively, and feel that they are reflected in mathematics learning.

Vision and Goals of the Grade 9 Mathematics Course

The needs of learners are diverse, and all learners have the capacity to develop the knowledge, concepts, skills, and perspectives they need to become informed, productive, and responsible citizens in their own communities and within the world.

How mathematics is contextualized, positioned, promoted, discussed, taught, learned, evaluated, and applied affects the learning experiences and academic outcomes of all students. Mathematics can be appreciated for its innate beauty, as well as for its role in making sense of the world. Having a solid foundation in, a deep appreciation for, and excitement about mathematics, as well as recognizing their identities, lived experiences, and communities in their mathematics learning, will help ensure that all students grow more confident and capable as they step into the future.

All students bring their mathematical experiences from various contexts to school. Educators can value and build on these lived experiences so that mathematics classrooms become spaces that honour diverse mathematical ideas and thoughts, and incorporate multiple ways of knowing and doing. Such spaces allow all students to become flexible and adaptive learners in an ever-changing world.

The vision of this mathematics course is to support all students as they develop healthy and strong identities as mathematics learners and grow to be mathematically skilled, to enhance their ability to use mathematics to make sense of the world around them, and to enable them to make critical decisions while engaged in mathematical thinking. This vision is attained in a mathematics classroom filled with high academic expectations and deep engagement that generates enthusiasm and curiosity – an inclusive classroom where all students receive the highest-quality mathematics instruction and learning opportunities, are empowered to interact as confident mathematics learners, and are thereby supported in reaching their full potential.

The goal of the Ontario mathematics curriculum is to provide all students with the key skills required to:

- understand the importance of and appreciate the beauty and wonder of mathematics;
- recognize and appreciate multiple mathematical perspectives;
- make informed decisions and contribute fully to their own lives and to today's interconnected local and global communities;
- adapt to changes and synthesize new ideas;
- work both independently and collaboratively to approach challenges;
- communicate effectively;
- think critically and creatively to connect, apply, and leverage mathematics within other areas of study including science, technology, engineering, the arts, and beyond.

A strong foundation of mathematics is an important contributor to students' future success and an essential part of becoming an informed citizen. In order to develop a strong understanding of mathematics and the ability to apply mathematics in real life, all students must feel that they are connected to the curriculum – to what is taught, why it is taught, and how it is taught.

The Importance and Beauty of Mathematics

Mathematics is integral to every aspect of daily life – social, economic, cultural, and environmental. It is embedded into the rich and complex story of human history. People around the world have used, and continue to contribute, mathematical knowledge, skills, and attitudes to make sense of the world around them and to develop new mathematical thinking and appreciation for mathematics. Mathematics is conceptualized and practised in many different ways across diverse local and global cultural contexts. It is part of diverse knowledge systems composed of culturally situated thinking and practices. From counting systems, measurement, and calculation to geometry, spatial sense, trigonometry, algebra, functions, calculus, and statistics, mathematics has been evident in the daily lives of people and communities across human histories.

Today, mathematics is found all around us. For example, mathematics can be found in sports performance analysis, navigation systems, electronic music production, computer gaming, graphic art, quantum physics, climate change modelling, and so much more. Mathematics skills are necessary when we buy goods and services online, complete our taxes, do beading, construct buildings, and play sports. Mathematics also exists in nature, storytelling, music, dancing, puzzles, and games. Proficiency with mathematical ideas is needed for many careers, including but not limited to engineering, health care and medicine, psychology, computer science, finance, landscape design, fashion design, architecture, agriculture, ecology, the arts, the culinary arts, and many other skilled trades. In fact, in every field of pursuit, the analytical, problem-solving, critical-thinking, and creative-thinking skills that students develop through the study of mathematics are evident. In the modern age of evolving technologies, artificial intelligence, and access to vast sources of information and big data, knowing how to navigate, interpret, analyse, reason, evaluate, and problem solve is foundational to everyday life.

Mathematics can be understood as a way of studying and understanding structure, order, patterns, and relationships. The power of mathematics is evident in the connections among seemingly abstract mathematical ideas. The applications of mathematics have often yielded fascinating representations and results. As well, the aesthetics of mathematics have also motivated the development of new mathematical thinking. The beauty in mathematics can be found in the process of deriving elegant and succinct approaches to resolving problems.

At times, messy problems and seeming chaos may culminate in beautiful, sometimes surprising, results that are both simple and generalizable. Elegance and chaos are both integral to the beauty of mathematics itself and to the mathematical experience. In other words, the beauty of mathematics is illustrated and enhanced by students' diverse interpretations, strategies, representations, and identities – not diminished by them. Most importantly, students can experience wonder and beauty when they make exciting breakthroughs in problem solving. Therefore, these two aspects of mathematics, aesthetics and application, are deeply interconnected.

The Grade 9 mathematics course strives to equip all students with the knowledge, skills, and habits of mind that are essential to understanding and enjoying the importance and beauty of mathematics. Learning in Grade 9 mathematics begins with a focus on the fundamental concepts and foundational skills. This leads to an understanding of mathematical structures, operations, processes, and language that provides students with the means necessary for reasoning, justifying conclusions, and expressing and communicating mathematical ideas.

When educators put student learning at the centre, provide relevant and meaningful learning opportunities, and use technology strategically to enhance learning experiences, all students are supported as they learn and apply mathematical concepts and skills within and across strands and other subject areas.

The Grade 9 mathematics course emphasizes the importance of establishing an inclusive mathematical learning community where all students are invited to experience the living practice of mathematics, to work through challenges, and to find beauty and success in problem solving. As students engage with the curriculum, they are supported in incorporating their lived experiences and existing mathematical understandings, and then integrating the new ideas they learn into their daily lives. When students recognize themselves in what is taught and how it is taught, they begin to view themselves as competent and confident mathematics learners who belong to the larger mathematics community. As students develop mathematical knowledge and skills, they grow as mathematical thinkers. As students explore histories of mathematics and comprehend the importance and beauty of mathematics, they develop their mathematical agency and identity, at the same time as they make connections to other subjects and the world around them.

Human Rights, Equity, and Inclusive Education in Mathematics

Research indicates that there are groups of students (for example, Indigenous students, Black students, students experiencing homelessness, students living in poverty, students with LGBTQ+ identities, and students with special education needs and disabilities) who continue to experience systemic barriers to accessing high-level instruction in and support with learning mathematics. Systemic barriers, such as racism, implicit bias, and other forms of discrimination, can result in inequitable academic and life outcomes, such as low confidence in one's ability to learn mathematics, reduced rates of credit completion, and leaving the secondary school system prior to earning a diploma. Achieving equitable outcomes in mathematics for all students requires educators to be aware of and identify these barriers, as well as the ways in which they can overlap and intersect, which can compound their effect on student well-being, student success, and students' experiences in the classroom and in the school. Educators must not only know about these barriers, they must work actively and with urgency to address and remove them.

Students bring abundant cultural knowledges, experiences, and competencies into mathematical learning. It is essential for educators to develop pedagogical practices that value and centre students' prior learning, experiences, strengths, and interests. Such pedagogical practices are informed by and build on students' identities, lived experiences, and linguistic resources. When educators employ such pedagogy, they hold appropriate and high academic expectations of students, applying the principles of Universal Design for Learning and differentiated instruction to provide multiple entry points and maximize opportunities for all students to learn. By acknowledging and actively working to eliminate the systemic barriers that some students face, educators create the conditions for authentic experiences that empower student voices and enhance their sense of belonging, so that each student can develop a healthy identity as a mathematics learner and can succeed in mathematics and in all other subjects. Mathematics learning that is student-centred allows students to find relevance and meaning in what they are learning and to make connections between the curriculum and the world outside the classroom.

In mathematics classrooms, teachers also provide opportunities for cross-curricular learning and for teaching about human rights. To create anti-racist, anti-discriminatory learning environments, all educators must be committed to equity and inclusion and to upholding and promoting the human rights of every learner. Students of all identities and social locations have the right to mathematics opportunities that allow them to succeed, personally and academically. In any mathematics classroom, it is crucial to acknowledge students' intersecting social identities and their connected lived realities. Educators have an obligation to develop and nurture learning environments that are reflective of and responsive to students' strengths, needs, cultures, and diverse lived experiences – identity-affirming learning environments free from discrimination. In such learning environments, educators set appropriate and high academic expectations for all.

Culturally Responsive and Relevant Pedagogy in Mathematics

High-quality instruction that emphasizes deep mathematical thinking and cultural and linguistic knowledge and that addresses issues of inequity is the foundation of culturally responsive and relevant pedagogy (CRRP) in mathematics. In CRRP classrooms, teachers reflect on their own identities and pay attention to how those identities affect their teaching, their ideas, and their biases. Teachers also learn about students' identities, identifications, and/or affiliations and connected lived experiences. Teachers develop an understanding of how students are thinking about mathematical concepts according to their cultural backgrounds and experiences, and make connections with these cultural ways of knowing in their pedagogy. This approach to pedagogy develops social consciousness and critique while valorizing students' cultural backgrounds, communities, and cultural and linguistic competences. Teachers build on students' experiences, ideas, questions, and interests to support the development of an engaging and inclusive mathematics classroom community.

In mathematics classrooms, educators use CRRP to create teaching and learning opportunities to engage students in shaping much of the learning and to promote mathematical agency investment in the learning. When students develop agency, they are motivated to take ownership of their learning of, and progress in, mathematics. Teaching about diverse mathematical approaches and figures in history, from different global contexts, can offer opportunities for students to feel that they are reflected in mathematical learning – a key factor in developing students' sense of self – and to learn about others, and about the multiple ways mathematics exists in all aspects of the world around them.

Mathematics is situated and produced within cultures and cultural contexts. The curriculum is intended to expand historical understanding of the diversity of mathematical thought. In an anti-racist and anti-discriminatory environment, teachers know that there is more than one way to develop a solution, and students are exposed to multiple ways of knowing and encouraged to explore multiple ways of finding answers.

Indigenous pedagogical approaches emphasize holistic, experiential learning, teacher modelling, and the use of collaborative and engaging activities. Teachers differentiate instruction and assessment opportunities to encourage different ways of learning, to allow students to learn from and with each other, and to promote an awareness of and respect for the diverse and multiple ways of knowing that are relevant to and reflective of students' lived experiences in classrooms, schools, and the world. When making connections between mathematics and real-life applications, teachers are encouraged to work in partnership with First Nations, Inuit, and Métis individuals, communities, and/or nations. Teachers may respectfully incorporate culturally specific examples that highlight First Nations, Inuit, and Métis cultures, histories, present-day realities, ways of knowing, and contributions, to infuse Indigenous knowledges and perspectives meaningfully and authentically into the mathematics program. In this way, culturally specific examples centre Indigenous students as mathematical thinkers, and strengthen learning and course content so that all students continue to learn about diverse cultures and communities in a respectful and informed way. Students' mind, body, and spirit are nourished through connections and creativity.

More information on equity and inclusive education can be found in the "Human Rights, Equity, and Inclusive Education" subsection of "Considerations for Program Planning".

Principles Underlying the Grade 9 Mathematics Curriculum

- **A mathematics curriculum is most effective when it values and honours the diversity that exists among students and within communities.**
  The Grade 9 mathematics curriculum is based on the belief that all students can and deserve to be successful in mathematics. In particular, an inclusive curriculum is built on the understanding that not all students necessarily learn mathematics in the same way, use the same resources (e.g., tools and materials), or learn within the same time frames. Setting high academic expectations and building a safe and inclusive community of learners requires the purposeful use of a variety of instructional and assessment strategies and approaches that build on students' prior learning and experiences, and create an optimal and equitable environment for mathematics learning. The curriculum emphasizes the need to eliminate systemic barriers and to serve students belonging to groups that have been historically disadvantaged and underserved in mathematics education.
- **A robust mathematics curriculum is essential for ensuring that all students reach their full potential.**
  The Grade 9 mathematics curriculum challenges all students by including learning expectations that build on students' prior knowledge and experience; involve higher-order thinking skills; and require students to make connections between their lived experiences, mathematical concepts, other subject areas, and situations outside of school. This learning enables all students to gain a powerful knowledge of the usefulness of the discipline and an appreciation of the histories and importance of mathematics.
- **A mathematics curriculum provides all students with the fundamental mathematics concepts and foundational skills they require to become capable and confident mathematics learners.**
  The Grade 9 mathematics curriculum provides a balanced approach to the teaching and learning of mathematics. It is based on the belief that all students learn mathematics most effectively when they can build on prior knowledge to develop a solid understanding of the concepts and

skills in mathematics, and when they are given opportunities to apply these concepts and skills as they solve increasingly complex tasks and investigate mathematical ideas, applications, and situations in everyday contexts. As students continue to explore the relevance of mathematics, they further develop their identity and agency as capable mathematics learners.

- **A progressive mathematics curriculum includes the strategic integration of technology to support and enhance the learning and doing of mathematics.**
  The Grade 9 mathematics curriculum strategically integrates the use of appropriate technologies to support all students in developing conceptual understanding and procedural fluency, while recognizing the continuing importance of students' mastering the fundamentals of mathematics. For some students, assistive technology also provides an essential means of accessing the mathematics curriculum and demonstrating their learning. Students develop the ability to select appropriate tools and strategies to perform particular tasks, to investigate ideas, and to solve problems. The curriculum sets out a framework for learning important skills, such as problem solving, coding, and modelling, as well as opportunities to develop critical data literacy, information literacy, and financial literacy skills.
- **A mathematics curriculum acknowledges that the learning of mathematics is a dynamic, gradual, and continuous process, with each stage building on the last.**
  The Grade 9 mathematics curriculum is dynamic, continuous, and coherent and is designed to support all students in developing an understanding of the interconnected nature of mathematics. Students come to understand how concepts develop and how they build on one another. As students communicate their reasoning and findings, they move towards new understandings. Teachers observe and listen to all students and then responsively shape instruction in ways that foster and deepen student understanding of important mathematics. The fundamental concepts, skills, and processes introduced in the elementary grades support students in extending their learning in the secondary grades.
- **A mathematics curriculum is integrated with the world beyond the classroom.**
  The Grade 9 mathematics curriculum provides opportunities for all students to investigate and experience mathematical situations they might find outside the classroom and develop an appreciation for the beauty and wide-reaching nature and importance of mathematics. The overall curriculum integrates and balances concept development and skill development, including social-emotional learning skills, as well as the use of mathematical processes and real-life applications.
- **A mathematics curriculum motivates students to learn and to become lifelong learners.**
  The Grade 9 mathematics curriculum is brought to life in the classroom, where students develop mathematical understanding and are given opportunities to connect their knowledge and skills to wider contexts and other disciplines. Making connections to the world around them stimulates their interest and motivates them to become lifelong learners with healthy attitudes towards mathematics. Teachers bring the mathematics curriculum to life using their knowledge of:
  - the mathematics curriculum;
  - the backgrounds and identities of all students, including their past and ongoing experiences with mathematics and their learning strengths and needs;
  - mathematical concepts and skills, and the ways in which they are connected across the strands, other grades, other disciplines, and the world outside the classroom;

- instructional approaches and assessment strategies best suited to meet the learning needs of each student;
- resources designed to support and enhance the achievement of and engagement with the curriculum expectations, while fostering an appreciation for and joy in mathematics learning.

Roles and Responsibilities

Students

It is essential that all students continue to develop a sense of responsibility for and ownership of their own learning as they begin their journey through secondary school. Mastering the skills and concepts connected with learning in the mathematics curriculum requires a commitment to:

- continual and consistent personal reflection and goal setting;
- a belief that they are capable of succeeding in mathematics;
- developing the skills to persevere when taking on new challenges;
- connecting prior experiences, knowledge, skills, and habits of mind to new learning;
- a willingness to work both independently and collaboratively in an inclusive environment;
- dedication to ongoing practice;
- a willingness and an ability to receive and respond to meaningful feedback and ask questions to clarify understanding;
- a willingness to explore new learning in mathematics and share insights and experiences.

Through ongoing practice and reflection, all students can develop a strong and healthy mathematical identity whereby they value and appreciate mathematics as a discipline, feel themselves to be confident and competent mathematics learners, and understand what successful mathematics learning and being an effective mathematician look like.

Students' experiences influence their attitudes towards mathematics education and can have a significant impact on their engagement with mathematics learning and their subsequent success in achieving the expectations. Students who are engaged in their learning and who have opportunities to solve interesting, relevant, and meaningful problems within a supportive and inclusive learning environment are more likely to adopt practices and behaviours that support mathematical thinking. More importantly, they are more likely to be successful in their learning, which contributes to their enjoyment of mathematics and increases their desire to pursue further mathematics learning.

With teacher support and encouragement, students learn that they can apply the skills they acquire in mathematics to other contexts and subjects. For example, they can apply the problem-solving skills they develop in mathematics to their study of the science and Canadian and world studies curricula. They can also make connections between their learning and life beyond the classroom. For example, when presented with an issue or a contextually relevant STEM-based (science, technology, engineering, and mathematics–based) problem, they can look for potential applications of mathematical modelling. They can also begin to identify how mathematical modelling can be used to answer important questions related to global health, the environment, and sustainable, innovative development, or to address various issues that are relevant to their lives and communities.

Parents

Parents[1] are significant role models for their children and play an integral part in their children's experiences with mathematics. It is important for schools and parents, and in some situations, caring and trusted adults in students' lives who are not their parents, to work together to ensure that they provide a mutually supportive framework for young people's mathematics education. Research assures us of the positive impact of parent engagement and parent-child communication about mathematics on student success.

Parents can play a role in their children's success by speaking positively about mathematics and modelling the attitude that mathematics is enjoyable, worthwhile, and valuable. By encouraging their children to acknowledge challenges, to persevere when solving problems, and to believe that they can succeed in mathematics, parents help them build self-confidence and a sense of identity as mathematics learners.

Parents can support their children's mathematics success by showing an interest in what their children are learning. Parents are encouraged to engage with mathematics alongside their children by asking about their experiences in class and by finding ways to apply what is being learned in class to everyday contexts. Mathematics is everywhere, and parents can help their children make connections between what they are learning at school and everyday experiences at home and in the community, using tasks such as making appropriate choices when shopping, or saving for future needs. Parents can include their children in the things they do themselves that involve mathematics, such as estimating the amount of material needed to redecorate or renovate a room, or the quantities of ingredients needed to cook a meal. Through family activities, such as enjoying mathematics-based puzzles and games, making crafts, and beading jewelry together, parents can create opportunities for mental mathematics estimations and calculations and for making predictions. Parents can support their children's learning by encouraging them to complete their mathematics tasks, to practice new skills and concepts, to apply new mathematics learning to experiences at home, and to connect mathematical experiences at home to learning at school.

As students begin their journey through secondary school, parents can help them consider how mathematics may play a role in their future by talking about education and career goals or connecting with community partners to gather information. Parents can help their children make connections

[1] The word *parent(s)* is used on this website to refer to parent(s) and guardian(s). It may also be taken to include caregivers or close family members who are responsible for raising the child.

between what they are learning, potential careers, and their future choice of postsecondary pathways – such as apprenticeship, skilled trades, community living, college, university, or the workplace.

Schools offer a variety of opportunities for parents to learn more about how to support their children's mathematics learning: for example, events related to mathematics may be held at the school; teachers may provide newsletters or communicate with parents through apps or social media; and school or board websites may provide helpful tips about how parents can engage in their children's mathematics learning outside of school and may even provide links where they can learn more or enjoy mathematics-related activities together.

If parents need more information about what their children are learning, and how to support their children's success in mathematics, teachers are available to answer questions and provide information and resources.

Teachers

Teachers have the most important role in the success of students in mathematics. Teachers are responsible for ensuring that all students receive the highest quality of mathematics education. This requires them to have high academic expectations of all students, provide appropriate supports for learning, and believe that all students are capable math learners. Teachers bring expertise and skills to providing varied and equitable instructional and assessment approaches to the classroom. Teachers plan a mathematics program using an asset-based approach that affirms students' identities, reflects their lived experiences, leverages their strengths, and addresses their needs in order to ensure equitable, accessible, and engaging learning opportunities for every student. The attitude with which teachers themselves approach mathematics is critical, as teachers are important role models for students.

Teachers place students' well-being and academic success at the centre of their mathematics planning, teaching, and assessment practices, and understand how the learning experiences they provide will develop an appreciation of mathematics and foster a healthy attitude and engagement in all students. Teachers have a thorough understanding of the mathematics content they teach, which enables them to provide relevant and responsive, high-quality mathematical opportunities through which all students can develop their understanding of mathematical knowledge, concepts, and skills. Teachers understand the learning continua along which students develop their mathematical thinking and, with effective use of direct instruction and high-quality mathematical tasks, can thus support all students' movement along these continua. Teachers provide ongoing meaningful feedback to all students about their mathematics learning and achievement, which helps to build confidence and provide focused next steps. Teachers support students in developing their ability to solve problems, reason mathematically, and connect the mathematics they are learning to the real world around them. They recognize the importance of emphasizing and illustrating the usefulness of mathematics in students' lives, and of integrating mathematics with other areas of the curriculum – such as making connections with science, engineering, art, and technology to answer scientific questions or solve problems, or engaging in political debate and community development. They recognize the importance of supporting students in learning about careers involving mathematics, and of supporting the development of students' mathematical agency to grow their identity as capable mathematical thinkers.

As part of effective teaching practice, teachers use multiple ways and both formal and informal means to communicate with parents and develop partnerships between home or caring adults and school that meet the varied needs of families. Through various types of communication, teachers discuss with parents or caring adults what their children are learning in mathematics at school. These communications also help teachers better understand students' mathematical experiences beyond the classroom, and learn more about students' interests, skills, and aspirations. Ongoing communication leads to stronger connections between the home, community, and school to support student learning and achievement in mathematics.

Principals

Principals model the importance of lifelong learning and understand that mathematics plays a vital role in the future success of students. Principals provide instructional leadership for the successful implementation of the mathematics curriculum – in the school and in communications with parents – by emphasizing the importance of a well-planned mathematics program and high-quality mathematical instruction, by promoting the idea that all students are capable of becoming confident mathematics learners, and by encouraging a positive and proactive attitude towards mathematics and student agency in mathematics.

Principals work in partnership with teachers and parents to ensure that all students have access to the best possible educational experience. To support student learning, principals monitor the implementation of the Ontario mathematics curriculum. Principals ensure that English language learners are being provided the accommodations and/or modifications they require for success in the mathematics program. Principals are also responsible for ensuring that every student who has an Individual Education Plan (IEP) is receiving the modifications and/or accommodations described in their plan – in other words, for ensuring that the IEP is properly developed, implemented, and monitored.

Ensuring that teachers have the competence, agency, support, confidence, resources, and tools they need to deliver a high-quality program is essential. Principals collaborate with teachers and school and system leaders to develop professional learning opportunities that deepen teachers' curriculum knowledge, mathematical content knowledge for teaching, and pedagogy, and enhance their self-efficacy in teaching mathematics.

Community Partners

Community partners are an important resource for a school's mathematics education program. Community partners can also contribute to the success of the program by providing support for families, children and youth, and educators, so that they in turn may support student learning. Relationships with local businesses, volunteer groups, Indigenous communities, postsecondary institutions, informal learning spaces such as museums and science centres, and community organizations such as those that serve newcomer families or marginalized communities, can provide opportunities for authentic perspectives and real-world application of mathematics, as well as support for families. Nurturing partnerships with other schools can facilitate the sharing of resources, strategies, and facilities, the development of professional learning opportunities for staff, and the hosting of special events such as mathematics or coding workshops for students.

Communities provide social contexts for learning, such as opportunities for volunteer work or employment for students at the secondary level. Students bring knowledge and experiences from their homes and communities that are powerful resources in creating productive learning environments. By involving members of the community, teachers and principals can position mathematics learning as collaborative and experiential. Membership in a community also supports students in developing a sense of belonging and in building their identity as mathematics learners in relation to, and with, others.

Elements of the Grade 9 Mathematics Course

Overview

The Grade 9 mathematics course builds on the elementary program and is based on the same fundamental principles.

The overall aim of the Grade 9 mathematics course is to ensure that all students can access any secondary mathematics course they need in order to pursue future studies and careers that are of interest to them.

This course is designed to be inclusive of all students in order to facilitate their transition from the elementary grades to the secondary level. It offers opportunities for all students to build a solid foundation in mathematics, broaden their knowledge and skills, and develop their mathematical identity. This approach allows students to make informed decisions in choosing future mathematics courses based on their interests, and in support of future plans for apprenticeship training, university, college, community living, or the workplace.

Similar to the elementary curriculum, the Grade 9 course adopts a strong focus on the processes that best enable students to understand mathematical concepts and learn related skills. Attention to the mathematical processes is considered essential to a balanced mathematics program. The seven mathematical processes identified in the curriculum include *problem solving*, *reasoning and proving*, *reflecting*, *connecting*, *communicating*, *representing*, and *selecting tools and strategies*.

Throughout the course, students actively participate in the learning of mathematics by making connections to their lived experiences and to real-life applications. They continue to develop critical consciousness of how socio-cultural structures within systems impact individual experiences and opportunities, and to shape their identities as mathematics learners.

Teachers implement the curriculum through effective assessment and instructional practices that are rooted in Culturally Responsive and Relevant Pedagogy. Teachers utilize a variety of assessment and instructional approaches that provide students with multiple entry points to access mathematics learning and multiple opportunities to demonstrate their achievement in mathematics.

This course continues the learning from Grade 8 and prepares students for success in all senior secondary mathematics courses in all pathways moving forward. Students who successfully complete the Grade 9 mathematics course may proceed to a mathematics course in Grade 10.

The following section is in effect for the 2021–22 school year and will be updated as the secondary mathematics program is revised. The 2005 Mathematics curriculum for Grade 10 and the 2007 Mathematics curriculum for Grades 11–12 remain in effect. All references to Grade 9 that appear in *The Ontario Curriculum, Grades 9 and 10: Mathematics, 2005* and *The Ontario Curriculum, Grades 11 and 12: Mathematics, 2007* have been superseded by the section below.

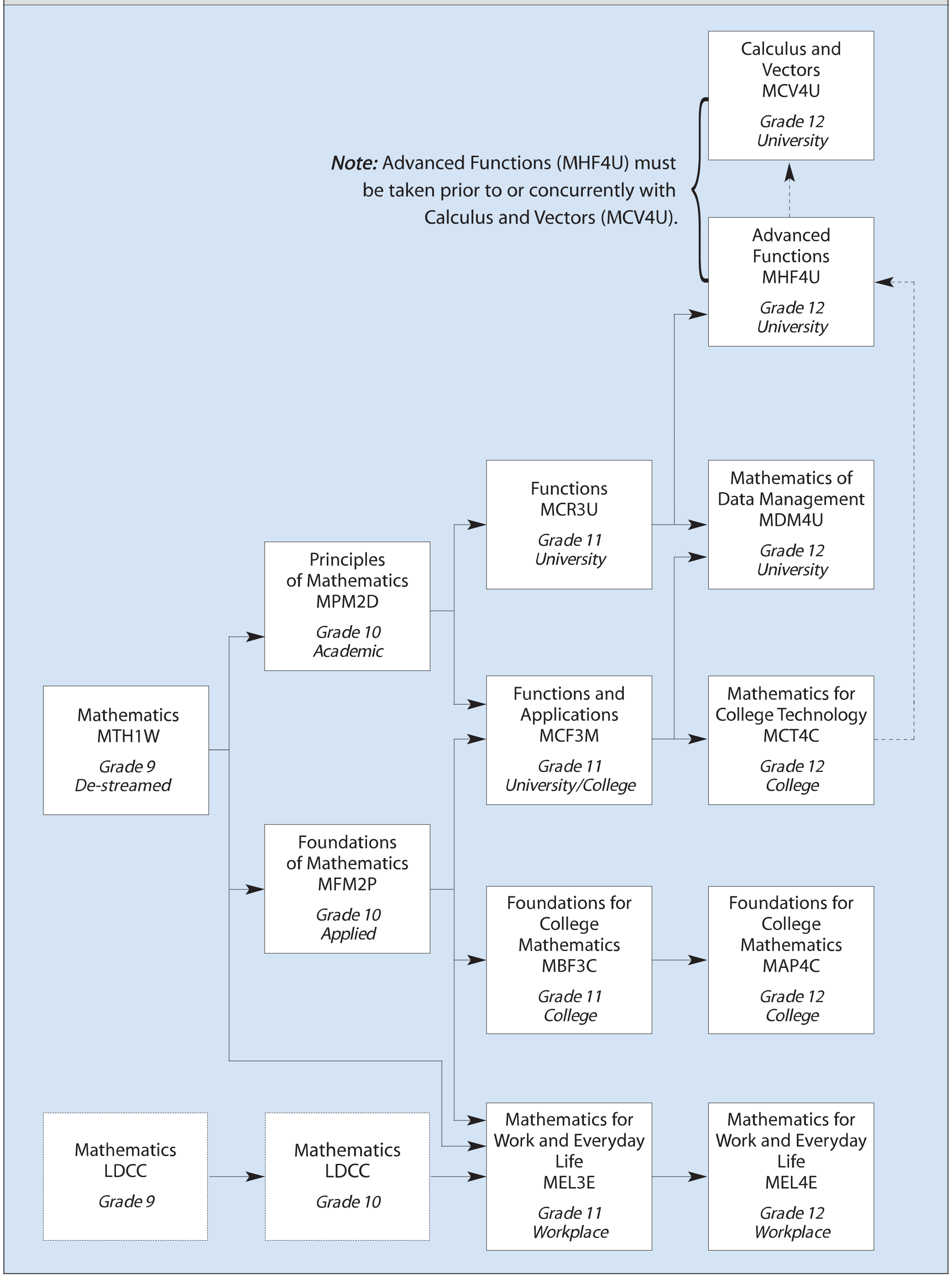
Courses in Mathematics, Grades 9 to 12

Grade	Course Name	Course Type	Course Code	Credit Value	Prerequisite
9	Mathematics	De-streamed	MTH1W	1.0	None
10	Principles of Mathematics	Academic	MPM2D	1.0	Grade 9 Mathematics, De-streamed (2021), or Grade 9 Principles of Mathematics, Academic (2005)
10	Foundations of Mathematics	Applied	MFM2P	1.0	Grade 9 Mathematics, De-streamed (2021), or Grade 9 Foundations of Mathematics, Applied (2005)
11	Functions	University	MCR3U	1.0	Grade 10 Principles of Mathematics, Academic
11	Functions and Applications	University/ College	MCF3M	1.0	Grade 10 Principles of Mathematics, Academic, or Grade 10 Foundations of Mathematics, Applied
11	Foundations for College Mathematics	College	MBF3C	1.0	Grade 10 Foundations of Mathematics, Applied
11	Mathematics for Work and Everyday Life	Workplace	MEL3E	1.0	Grade 9 Mathematics, De-streamed (2021), or Grade 9 Principles of Mathematics, Academic (2005), or Grade 9 Foundations of Mathematics, Applied (2005), or a Grade 10 Mathematics LDCC (locally developed compulsory credit) course
12	Advanced Functions	University	MHF4U	1.0	Grade 11 Functions, University, or Grade 12 Mathematics for College Technology, College
12	Calculus and Vectors	University	MCV4U	1.0	Grade 12 Advanced Functions, University, must be taken prior to or concurrently with Calculus and Vectors

12	Mathematics of Data Management	University	MDM4U	1.0	Grade 11 Functions, University, or Grade 11 Functions and Applications, University/College
12	Mathematics for College Technology	College	MCT4C	1.0	Grade 11 Functions and Applications, University/College, or Grade 11 Functions, University
12	Foundations for College Mathematics	College	MAP4C	1.0	Grade 11 Foundations for College Mathematics, College, or Grade 11 Functions and Applications, University/College
12	Mathematics for Work and Everyday Life	Workplace	MEL4E	1.0	Grade 11 Mathematics for Work and Everyday Life, Workplace

Prerequisite Chart for Mathematics, Grades 9 – 12

This chart maps out all the courses in the discipline and shows the links between courses and the possible prerequisites for them. It does not attempt to depict all possible movements from course to course.



Note: LDCC – locally developed compulsory credit course (LDCC courses are not outlined in this curriculum.)

Note: For students who completed any of the Grade 9 mathematics courses prior to September 2021, refer to the prerequisite chart on page 10 of *The Ontario Curriculum, Grades 11 and 12: Mathematics, 2007*.

Locally Developed Compulsory Credit Courses (LDCCs)

School boards may offer up to two locally developed compulsory credit courses in mathematics – a Grade 9 course and/or a Grade 10 course – that may be used to meet the compulsory credit requirement in mathematics for one or both of these grades. The locally developed Grade 9 and/or Grade 10 compulsory credit courses prepare students for success in the Grade 11 and Grade 12 workplace preparation courses.

Half-Credit Courses

The course outlined in this curriculum is designed to be offered as a full-credit course. However, it may also be delivered as two half-credit courses. Half-credit courses, which require a minimum of fifty-five hours of scheduled instructional time, must adhere to the following conditions:

- The two half-credit courses created from a full course must together contain all of the expectations of the full course.
- The expectations for each half-credit course must be divided in a manner that best enables students to achieve the required knowledge and skills in the allotted time.
- A course that is a prerequisite for another course in the secondary curriculum may be offered as two half-credit courses, but students must successfully complete both parts of the course to fulfil the prerequisite. (Students are not required to complete both parts unless the course is a prerequisite for another course they wish to take.)
- The title of each half-credit course must include the designation Part 1 or Part 2. A half credit (0.5) will be recorded in the credit-value column of both the report card and the Ontario Student Transcript.

Boards will ensure that all half-credit courses comply with the conditions described above, and will report all half-credit courses to the ministry annually in the School October Report.

Curriculum Expectations for the Grade 9 Mathematics Course

The expectations identified for this course describe the knowledge, concepts, and skills that students are expected to acquire, demonstrate, and apply in their class work and tasks, on tests, in demonstrations, and in various other activities on which their achievement is assessed and evaluated.

Mandatory learning is described in the overall and specific expectations of the curriculum.

Two sets of expectations – overall expectations and specific expectations – are listed for each *strand*, or broad area of the curriculum. The strands in this course are lettered AA and A through F.

The *overall expectations* describe in general terms the knowledge and skills that students are expected to demonstrate by the end of the course. The *specific expectations* describe the expected knowledge, concepts, and skills in greater detail. The specific expectations are grouped under numbered subheadings, each of which indicates the strand and the overall expectation to which the group of specific expectations corresponds (e.g., "B2" indicates that the group relates to overall expectation 2 in strand B). This organization is not meant to imply that the expectations in any one group are achieved independently of the expectations in the other groups, nor is it intended to imply that learning the expectations happens in a linear, sequential way. The numbered headings are used merely as an organizational structure to help teachers focus on particular aspects of knowledge, concepts, and skills as they develop various lessons and learning activities for students. In the mathematics curriculum, additional subheadings are used within each group of expectations to identify the topics addressed in the strand.

The knowledge and skills described in the expectations in Strand A: Mathematical Thinking and Making Connections apply to all areas of course content and must be developed in conjunction with learning in strands B through F. Teachers should ensure that students develop the mathematics knowledge and skills in appropriate ways as they work to achieve the curriculum expectations in strands B through F. Students' application of the knowledge and skills described in Strand A must be assessed and evaluated as part of their achievement of the overall expectations in strands B through F.

Note: Strand AA: Social-Emotional Learning (SEL) Skills in Mathematics is an exception. It has a single overall expectation that is to be included in classroom instruction throughout the course, *but not in assessment, evaluation, or reporting*.

Teacher Supports

The expectations are accompanied by "teacher supports", which may include examples, key concepts, teacher prompts, instructional tips, and/or sample tasks. These elements are intended to promote understanding of the intent of the specific expectations and are offered as illustrations for teachers. *The teacher supports do not set out requirements for student learning; they are optional, not mandatory*.

"Examples" are meant to illustrate the intent of the expectation, the kind of knowledge, concepts, or skills, the specific area of learning, the depth of learning, and/or the level of complexity that the expectation entails.

"Key concepts" identify the central principles and mathematical ideas that underpin the learning in that specific expectation.

"Teacher prompts" are sample guiding questions and considerations that can lead to discussions and promote deeper understanding.

“Instructional tips” are intended to support educators in delivering instruction that facilitates student learning related to the knowledge, concepts, and skills set out in the expectations.

“Sample tasks” are developed to model appropriate practice for the course. They provide possible learning activities for teachers to use with students and illustrate connections between the mathematical knowledge, concepts, and skills. Teachers can choose to draw on the sample tasks that are appropriate for their classrooms, or they may develop their own approaches that reflect a similar level of complexity and high-quality mathematical instruction. Whatever the specific ways in which the requirements outlined in the expectations are implemented in the classroom, they must, wherever possible, be inclusive and reflect the diversity of the student population and the population of the province. When designing inclusive learning tasks, teachers reflect on their own biases and incorporate their deep knowledge of the curriculum, as well as their understanding of the diverse backgrounds, lived experiences, and identities of students. Teachers will notice that some of the sample tasks address the requirements of the expectation they are associated with and incorporate mathematical knowledge, concepts, or skills described in expectations in other strands of the course. Some tasks are cross-curricular in nature and will cover expectations in other disciplines in conjunction with the mathematics expectations.

The Mathematical Processes

Students learn and apply the mathematical processes as they work to achieve the expectations outlined in the curriculum. All students are actively engaged in applying these processes throughout the course.

The mathematical processes that support effective learning in mathematics are as follows:

- problem solving
- reasoning and proving
- reflecting
- connecting
- communicating
- representing
- selecting tools and strategies

The mathematical processes can be understood as the processes through which all students acquire and apply mathematical knowledge, concepts, and skills. These processes are interconnected. Problem solving and communicating have strong links to all of the other processes. A problem-solving approach encourages students to reason their way to a solution or a new understanding. As students engage in reasoning, teachers further encourage them to pose questions, make conjectures, and justify solutions, orally and in writing. The communication and reflection that occur before, during, and after the process of problem solving support students as they work to articulate and refine their thinking and to examine the problem they are solving from different perspectives. This opens the door to recognizing the range of strategies that can be used to arrive at a solution. By understanding how others solve a problem, students can begin to reflect on their own thinking (a process known as “metacognition”) and the thinking of others, as well as their own language use (a process known as “metalinguistic awareness”), and to consciously adjust their own strategies in order to make their solutions as efficient and accurate as possible.

The mathematical processes cannot be separated from the knowledge, concepts, and skills that students acquire throughout the course. All students problem solve, communicate, reason, reflect, and so on, as they develop the knowledge, the understanding of mathematical concepts, and the skills required in all strands.

Problem Solving

Problem solving is central to doing mathematics. By learning to solve problems and by learning *through* problem solving, students are given, and create, numerous opportunities to connect mathematical ideas and to develop conceptual understanding. Problem solving forms the basis of effective mathematics programs that place all students’ experiences and queries at the centre of mathematical learning. Therefore, problem solving should be the foundation of mathematical instruction. It is considered an essential process through which all students are able to achieve the expectations in mathematics and is an integral part of the Ontario mathematics curriculum.

Problem solving:

- increases opportunities for the use of critical thinking skills (e.g., selecting appropriate tools and strategies, estimating, evaluating, classifying, assuming, recognizing relationships, conjecturing, posing questions, offering opinions with reasons, making judgements) to develop mathematical reasoning;
- supports all students in developing their own mathematical identity;
- allows all students to use the varied mathematical knowledge and experiences they bring to school;
- supports all students in making connections among mathematical knowledge, concepts, and skills, and between situations inside and outside the classroom;
- has the potential to promote the collaborative sharing of ideas and strategies, and promotes talking about and interacting with mathematics;
- empowers students to use mathematics to address issues relevant to their lived realities;
- facilitates the use of creative-thinking skills when developing solutions and approaches;
- supports students in finding enjoyment in mathematics and becoming more confident in their ability to do mathematics.

Most importantly, when problem solving is done in a mathematical context relevant to students’ experiences and/or derived from their own problem posing, it furthers their understanding of mathematics and develops their mathematical agency.

Problem-Solving Strategies. Problem-solving strategies are methods that can be used to solve problems of various types. Common problem-solving strategies include the following: simulating; making a model, picture, or diagram; using concrete materials; looking for a pattern; guessing and checking; making an organized list; making a table or chart; solving a simpler version of the problem; working backwards; and using logical reasoning. Teachers can support all students as they develop their use of these strategies by engaging with solving various kinds of problems – instructional problems, routine problems, and non-routine problems. As students develop their repertoire over time, they become more confident in posing their own questions, more mature in their problem-solving skills, and more flexible in using appropriate strategies when faced with new problem-solving situations.

Reasoning and Proving

Reasoning and proving are integral to mathematics and involve students using their understanding of mathematical knowledge, concepts, and skills to justify their thinking. Proportional reasoning, algebraic reasoning, spatial reasoning, statistical reasoning, and probabilistic reasoning are all forms of mathematical reasoning. Students also use their understanding of numbers and operations, geometric properties, and measurement relationships to reason through solutions to problems. Students develop algebraic reasoning by generalizing understanding of numbers and operations, properties, and relationships between quantities. They develop functional thinking by generalizing patterns and non-numeric sequences and using inverse operations. Students may need to identify assumptions in order to begin working on a solution. Teachers can provide all students with learning opportunities where they must form mathematical conjectures and then test or prove them to verify whether they hold true. Initially, students may rely on the viewpoints of others to justify a choice or an approach to a solution. As they develop their own reasoning skills, they will begin to justify or prove their solutions by providing evidence.

Reflecting

Students reflect when they are working through a problem to monitor their thought process, to identify what is working and what is not working, and to consider whether their approach is appropriate or whether there may be a more effective approach. Students also reflect after they have solved a problem by considering the reasonableness of their answer and whether adjustments need to be made. Teachers can support all students as they develop their reflecting and metacognitive skills by asking questions that have them examine their thought processes. In an inclusive learning environment, students also reflect on their peers' thinking processes to further develop deep understanding. Students can also reflect on how their new knowledge can be applied to past and future problems in mathematics.

Connecting

Experiences that allow all students to make connections – to understand, for example, how knowledge, concepts, and skills from one strand of mathematics are related to those from another – will support students in grasping general mathematical principles. Through making connections, students learn that mathematics is more than a series of isolated skills and concepts and that they can use their learning in one area of mathematics to understand another, and to understand other disciplines. Recognizing the relationships between representations, concepts, and procedures also supports the development of deeper mathematical understanding. In addition, making connections between the mathematics they learn at school and its significance in their everyday lives supports students in deepening their understanding of mathematics and allows them to understand how useful and relevant it is in the world beyond the classroom.

Communicating

Communication is an essential process in learning mathematics. Students communicate for various purposes and for different audiences, such as the teacher, a peer, a group of students, the whole class, a community member or group, or their family. They may use oral, visual, written, or gestural communication. Students also acquire the language of mathematics and develop their communication skills, which includes expressing, understanding, and using appropriate mathematical terminology, symbols, conventions, and models, through meaningful interactions with each other.

For example, teachers can ask students to:

- illustrate their mathematical understanding in various ways, such as with diagrams and representations;
- share and clarify their ideas, understandings, and solutions;
- create and defend mathematical arguments;
- provide meaningful descriptive feedback to peers;
- pose and ask relevant questions.

Communication also involves active listening and responding mindfully with an awareness of socio-cultural contexts. Using Culturally Responsive and Relevant Pedagogy, teachers provide opportunities for all students to contribute to discussions about mathematics in the classroom. Effective classroom communication requires a supportive and inclusive environment in which all members of the class are invited to participate and are valued when they speak and when they question, react to, and elaborate on the statements of their peers and the teacher.

Representing

Students represent mathematical ideas and relationships and model situations using tools, pictures, diagrams, graphs, tables, numbers, words, and symbols. Some students may also be able to use other languages and/or digital and multimodal resources. Teachers recognize and value the variety of representations that students use, as each student may have different prior access to and experiences with mathematics. While encouraging student engagement and affirming the validity of their representations, teachers support students in reflecting on the appropriateness of their representations and refining them. Teachers support students as they make connections among various representations that are relevant to both the student and the audience they are communicating with, so that all students can develop a deeper understanding of mathematical concepts and relationships. All students are supported in using the different representations appropriately and as needed to model situations, solve problems, and communicate their thinking.

Selecting Tools and Strategies

Students develop the ability to select appropriate tools, technology, and strategies to perform particular mathematical tasks, to investigate mathematical ideas, and to solve problems.

Tools. All students should be encouraged to select and use tools to illustrate mathematical ideas. Students come to understand that making their own representations is a powerful means of building understanding and of explaining their thinking to others. Using tools supports students as they:

- identify patterns and relationships;
- make connections between mathematical concepts and between concrete and abstract representations;
- test, revise, and confirm their reasoning;
- remember how they solved a problem;
- communicate their reasoning to others, including by gesturing.

Technology. A wide range of technological and digital tools can be used in many contexts for students to interact with as they learn and extend concepts, and do mathematics.

Students can use:

- computers, calculators, probes, and computer algebra systems to perform complex operations; create graphs; and collect, organize, and display data;
- digital tools, apps, and social media to investigate mathematical concepts and develop an understanding of mathematical relationships;
- statistical software to manipulate, analyse, represent, sort, and communicate real-life data sets of all sizes;
- coding software to better understand the structures and relationships of mathematics;
- dynamic geometry software and online geometry tools to develop spatial sense;
- computer programs to represent and simulate mathematical situations (i.e., mathematical modelling);
- communications technologies to support and communicate their thinking and learning;
- computers, tablets, and mobile devices to access mathematical information available on the websites of organizations around the world in the language of instruction and/or other languages and to develop information literacy.

Developing the ability to perform mental computations is an important aspect of student learning in mathematics. Students must, therefore, use technology with discretion, when it makes sense to do so. When students use technology in their mathematics learning, they should apply mental computation, reasoning, and estimation skills to predict and check the reasonableness of answers.

Strategies. Problem solving often requires students to select an appropriate strategy. Students learn to use more efficient ways to reach a conclusion. For example, students can solve problems involving a linear relationship by extending a pattern using pictures, creating a table of values, or developing a general case and solving an equation. The selection of an appropriate strategy may also be based on feasibility. For example, students may choose to collect their own samples of data or access data collected in large amounts via computer programs.

Social-Emotional Learning (SEL) Skills in Grade 9 Mathematics

Building social-emotional learning (SEL) skills in a secondary classroom involves continuing the development of students' self-awareness, self-management, social awareness, relationship skills, and responsible decision-making.[2] In this course, the focus is on the mathematical context and giving students the tools they need for success in their future mathematical learning, as they learn the skills for:

- recognizing and identifying emotions that support mathematical learning;
- recognizing sources of stress that present challenges to mathematical learning;
- identifying resources and supports that aid perseverance in mathematical learning;
- building healthy relationships and communicating effectively in mathematics;
- developing a healthy mathematical identity through building self-awareness;
- developing critical and creative mathematical thinking.

In an anti-racist and anti-discriminatory learning environment, explicit instruction, practice, modelling, self-reflection, and reinforcement both inside and outside the classroom make a difference in development of these skills. As with all instruction, continual consideration must be given to how educational systems and institutions can communicate and understand more inclusive perspectives on experiencing and displaying emotions, respect, and professionalism. SEL skills cannot be taught without the context of systemic oppression and racism that many Ontario students navigate daily. Research has shown that educator bias can negatively affect the evaluation of social-emotional learning skills in relation to particular groups of students; for example, Indigenous students; Black and other racialized students; students with special education needs and disabilities; and students otherwise marginalized.

At the same time, there is strong evidence that teaching transformational social-emotional learning skills at school, when implemented in an anti-racist and anti-discriminatory, culturally responsive and relevant way, can contribute to students' overall health and well-being and to successful academic performance. Developing social-emotional learning skills also supports positive mental health, as well as students' ability to learn and experience academic success. Learning related to the overall expectation in this strand occurs in the context of learning related to the other six strands, and the focus is on intentional instruction only, not on assessment, evaluation, or reporting.

In order for SEL to be effective, teaching and learning approaches must consider and address the lived realities of students, including the ways in which educator biases affect students' experiences in the

[2] CASEL (Collaborative for Academic, Social, and Emotional Learning), *Evidence-Based Social and Emotional Learning Programs: CASEL Criteria Updates and Rationale* (Chicago, IL: Author, 2020).

classroom. Approaches to support SEL instruction must be mediated through authentic and respectful conversations about students' lived realities. These realities may include the inequities students negotiate inside and outside the classroom, educator biases that perpetuate systemic racism, historical and intergenerational trauma related to the education system, institutional and interpersonal discrimination, and harassment.

Human rights principles[3] and the Education Act identify the importance of creating a climate of understanding of, and mutual respect for, the dignity and worth of each person, so that each person can contribute fully to the development and well-being of their community. Human rights law guarantees a person's right to equal treatment in education. It requires educators and school leaders to actively prevent all discrimination and harassment and respond appropriately when they do occur, to create an inclusive environment, to remove barriers that limit the ability of students, and to provide accommodations where necessary.

Intentional Instruction

Social-emotional learning skills can be developed across all subjects of the curriculum – including mathematics – as well as during various school activities, at home, and in the community. These skills support students in understanding and applying mathematical thinking and making connections across the course that are key to learning and doing mathematics. They support all students – and indeed all learners, including educators and parents – as they develop confidence, cope with challenges, and think critically. This in turn enables students to improve and demonstrate mathematics knowledge, concepts, and skills in a variety of situations. Social-emotional learning skills support every student in developing a healthy identity as a capable mathematics learner.

Educator self-reflection on their own socio-cultural awareness is an essential component in the instruction of SEL in Ontario schools. Self-reflection is an important part of understanding oneself, one's identity and worldview, one's own beliefs, one's unconscious biases, one's privilege, and one's responses to these. For educators, self-awareness and self-reflection help to interrogate and understand their own position, as well as provide some grounding principles that can be used to support all students in enhancing their social-emotional learning skills while teaching in a way that is culturally responsive. Ensuring a culturally responsive and reflective approach that supports students in developing social-emotional learning skills begins with educator reflection and consideration of the learning environment. Educators reflect on instructional strategies, classroom climate, and the cultural context in which they teach, and consider making adjustments in any of these areas to more effectively support student learning and well-being for all students. SEL skills are developed within a learning context and with consideration of the individual student, and of their relationships to the classroom teacher, peers, other educators, the larger school community, and the world beyond.

Working with students to identify their personal learning goals related to SEL skills ensures that the intended learning is clear and transparent to all students and that all lived experiences are recognized. For example, when teachers are explicitly teaching skills for healthy relationships during problem-solving

[3] More information on human rights in Ontario education is available in "Human Rights, Equity, and Inclusive Education" in the main "Considerations for Program Planning" section.

in mathematics, students and teachers work together to identify what these skills can look like and sound like. This may include recognizing different approaches to problem-solving that may be used in the students’ homes or communities and in a variety of cultures; using encouraging words when communicating; and listening to each other about using different problem-solving approaches if the first one doesn’t succeed. Educators model and teach these skills during instruction. Students may show their understanding of these skills in a variety of ways and reflect on their own progress individually.

Social-Emotional Learning Skills: Key Components and Sample Strategies

The chart below provides information about social-emotional learning skills, including key components and sample strategies in the context of mathematics learning.

Skills *What are the skills? How do they help? What do they look like in mathematics?*	**Key Components and Sample Strategies**

Recognizing and Identifying Emotions That Support Mathematical Learning
 	
Students often experience a range of emotions over the course of their day at school. They may feel happy, sad, angry, frustrated, or excited, or any number of emotions in combination. Students may struggle to identify and appropriately express their feelings. Learning to recognize different emotions can support students in interacting with mathematical content and within mathematical learning communities in healthy ways. When students understand the influence of thoughts and emotions on behaviour, they can improve the quality of their interactions and are better able to respond to themselves and others in ways that are compassionate and caring, and that honour their own social and emotional needs. In mathematics, as they learn new mathematics concepts and interact with others while problem solving, students have many opportunities to develop awareness of their emotions and to use communication skills to express their feelings and to respond with care when they recognize emotions in others.
	• Recognizing a range of emotions in self and others
	• Understanding connections between thoughts, feelings, and actions and the impacts of each of these on the others
	• Recognizing that new or challenging learning may involve a sense of excitement or an initial sense of discomfort
	• Applying strategies such as:
	    • identifying, naming, and reasoning through the cause of particular emotions
	    • using language such as “I’m feeling frustrated because…”
	    • using tools (e.g., pictures) and language to gauge intensity of emotion

Recognizing Sources of Stress That Present Challenges to Mathematical Learning
 	
Every day, students are exposed to a range of challenges that can contribute to feelings of stress. As they learn stress management and coping skills, they come to recognize that stress is a part of learning and life and that it can be managed. While taking steps to dismantle systemic barriers to student well-being and success, educators can support students as they learn ways to respond to challenges in mathematics learning that enable them to "bounce back" and, in this way, build resilience in the face of life's obstacles. Over time, with support, practice, feedback, reflection, and experience, students begin to build a set of personal coping strategies that they can carry with them through life. In mathematics, students work through challenging problems, understanding that their resourcefulness in using coping strategies strengthens their personal resilience.
	• Seeking support from peers, teachers, family, or their extended community
	• Applying strategies such as:
	  ◦ "chunking" a task or problem into manageable components and tackling one piece at a time
	  ◦ thinking of a similar problem
	  ◦ engaging in guided imagery and visualization
	  ◦ stretching
	  ◦ pausing and reflecting
	  ◦ using an iterative approach to solve a problem, including reframing questions, trying out different methods, estimating, and guessing and checking solutions

Identifying Resources and Supports That Aid Perseverance in Mathematical Learning
 	
In a supportive and inclusive environment, students have regular opportunities to practise and apply perseverance skills as they solve mathematical problems and develop an appreciation for learning from mistakes as a part of the mathematics learning process. Educators can support students in approaching challenges in life with an understanding that there is struggle in most successes and that accessing the right support can lead to success. To that end, students need to identify and access educators as key resources. Through regular interactions, students and educators can build relationships based on trust and respect. Educators can also support students in noticing and naming harmful classroom interactions such as microaggressions and discrimination, and can support them when they report incidents of harm. While building skills for perseverance can have an impact on an individual student level, it is important to recognize the critical role educators play when they actively take steps to acknowledge and address systemic barriers at all levels (in the classroom, in the school, across the system, in the community) that hinder mathematical learning for students.
	• Embracing mistakes as a necessary and helpful part of learning
	• Noticing strengths and positive aspects of experiences, appreciating the value of practice
	• Creating a list of supports and resources, including people, that can aid them in persevering
	• Applying strategies such as:
	  ◦ supporting peers by encouraging them to keep trying if they make a mistake
	  ◦ using personal affirmations like "I can do this."

Building Healthy Relationships and Communicating Effectively in Mathematics
 	
Healthy relationships are at the core of developing and maintaining physically and mentally safe, healthy, equitable, caring school and classroom communities. When students interact in meaningful ways with others, mutually respecting diversity of thought and expression, their sense of belonging within the school and community is enhanced. Learning healthy relationship skills helps students establish patterns of effective communication and inspires healthy, cooperative relationships. These skills include the ability to understand and appreciate another person's perspective, to empathize with others, to listen attentively, and to resolve conflict in healthy ways. In mathematics, students have opportunities to develop and practise skills that support healthy interaction with others as they work together in small groups or in pairs to solve math problems and confront challenges. Developing these skills helps students to communicate with teachers, peers, and family about mathematics with an appreciation of the beauty and wonder of mathematics.
	• Recognizing and understanding the impact of one's emotions and actions on others
	• Listening attentively
	• Considering other ideas and perspectives
	• Practicing empathy and care
	• Using conflict-resolution skills
	• Using cooperation and collaboration skills
	• Applying strategies such as:
	    ◦ seeking opportunities to help others
	    ◦ working as part of a team and playing different roles (e.g., leader, scribe or illustrator, data collector, observer) that contribute to outcomes in different ways

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<td>Developing a Healthy Mathematical Identity Through Building Self-Awareness

Knowing who we are and having a sense of purpose and meaning in our lives enables us to function in the world as self-aware individuals. Our sense of identity enables us to make choices that support our well-being and allows us to connect with and have a sense of belonging in various cultural and social communities. Educators should note that for First Nations, Métis, and Inuit students, the term “sense of identity and belonging” may also mean belonging to and identifying with a particular community and/or nation. Self-awareness and identity skills supports students in exploring who they are – their strengths, preferences, interests, values, and ambitions – and how their social and cultural contexts have influenced them. This exploration is grounded in affirming cultural heritage, considering social identities, and assessing the impact of beliefs and biases. In mathematics, as they learn new skills, students use self-awareness skills to monitor their progress and identify their individual strengths and gifts; in the process, they build their identity as mathematics learners who are capable of actualizing their individual pathways.</td>
<td>Knowing oneself
Caring for oneself
Having a sense of mattering and of purpose
Identifying personal strengths
Having a sense of belonging and community
Communicating their thinking and feelings about mathematics
Applying strategies such as:
building their identity as a math learner as they learn independently as a result of their efforts and challenges
monitoring progress in skill development
reflecting on strengths and accomplishments and sharing these with peers or caring adults</td>
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<td>Developing Critical and Creative Mathematical Thinking

Critical and creative thinking skills enable us to make informed judgements and decisions on the basis of a clear and full understanding of ideas and situations, and their implications, in a variety of settings and contexts. Students learn to question, interpret, predict, analyse, synthesize, detect bias, and distinguish between alternatives. They practise making connections, setting goals, creating plans, making and evaluating decisions, and analysing and solving problems for which there may be no clearly defined answers. In all aspects of the mathematics curriculum, students have opportunities to develop critical and creative thinking skills. Students have opportunities to build on prior learning, go deeper, and make personal connections through real-life applications.</td>
<td>Making connections
Making decisions
Evaluating choices, reflecting on and assessing strategies
Communicating effectively
Managing time
Setting goals and making plans
Applying organizational skills
Applying strategies such as:
determining what is known and what needs to be found
using various webs, charts, diagrams, and representations to help identify connections and interrelationships
using organizational strategies and tools, such as planners, trackers, and goal-setting frameworks</td>
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The Strands in the Grade 9 Mathematics Course

The Grade 9 mathematics course is designed to be inclusive of all students in order to facilitate their transition to learning at the secondary level by offering opportunities to broaden their knowledge and skills in mathematics. This approach allows students to make informed decisions in choosing future mathematics courses based on their interests and on requirements for future jobs, trades, and professions.

The Grade 9 mathematics course is organized into seven strands. Strand AA: Social-Emotional Learning (SEL) Skills in Mathematics focuses on a set of skills to be developed in the context of learning across all other strands. Strand A focuses on developing mathematical thinking and making connections to students' lived experiences as well as connecting curriculum to real-life applications as students acquire the mathematical concepts and skills set out in strands B through F. The remaining strands cover the interrelated content areas of number, algebra, data, geometry and measurement, and financial literacy. The Grade 9 mathematics course consolidates learning from the elementary grades and sets a foundation for learning in future secondary mathematics courses. The strands of the elementary mathematics program are closely aligned with those of the Grade 9 mathematics course, as shown in the following chart.

Elementary Mathematics	Grade 9 Mathematics
	AA. Social-Emotional Learning (SEL) Skills in Mathematics
A. Social-Emotional Learning (SEL) Skills in Mathematics and the Mathematical Processes	A. Mathematical Thinking and Making Connections
B. Number	B. Number
C. Algebra	C. Algebra
D. Data	D. Data
E. Spatial Sense	E. Geometry and Measurement
F. Financial Literacy	F. Financial Literacy

Strands in Grade 9 Mathematics
Strand AA: Social-Emotional Learning (SEL) Skills in Mathematics
Strand A: Mathematical Thinking and Making Connections • Mathematical processes • Making connections
Strand B: Number • Development and use of numbers • Number sets • Powers • Rational numbers • Applications
Strand C: Algebra • Development and use of algebra • Algebraic expressions and equations • Coding • Application of linear and non-linear relations • Characteristics of linear and non-linear relations
Strand D: Data • Application of data • Representation and analysis of data • Application of mathematical modelling • Process of mathematical modelling
Strand E: Geometry and Measurement • Geometric and measurement relationships
Strand F: Financial Literacy • Financial decisions

Strand AA: Social-Emotional Learning (SEL) Skills in Mathematics

This strand comprises a single overall expectation that is to be included in classroom instruction throughout the course, but not in assessment, evaluation, or reporting. Students are supported in exploring social-emotional learning skills in mathematics.

Strand A: Mathematical Thinking and Making Connections

Throughout the course, students apply the mathematical processes to develop conceptual understanding and procedural fluency while they engage in learning related to strands B through F. They also make connections between the mathematics they are learning and their lived experiences, various knowledge systems, and real-life applications, including employment and careers.

Strand B: Number

In this strand, students continue to make connections among various number systems, the cultural development of number concepts, and real-life applications. They will extend their learning about positive fractions, positive decimal numbers, and integers to work with negative fractions and negative decimal numbers. Students also extend their knowledge and skills related to percentages, ratios, rates, and proportions to make further connections to real life.

Strand C: Algebra

In this strand, students continue to develop an understanding of algebra by making connections between algebra and numbers as they generalize relationships with algebraic expressions and equations. Students will extend and apply coding skills to dynamically represent situations, analyse mathematics concepts, and solve problems in various contexts. Students will be introduced to various representations of linear and non-linear relations that they will study in more depth in future secondary mathematics courses. Students develop an understanding of constant rate of change and initial values of linear relations, and solve related real-life problems.

Strand D: Data

In this strand, students extend their data literacy skills to examine the collection, representation, and use of data, as well as their implications in various contexts. Students consolidate and extend their understanding of data involving one and two variables and its connections to real life. Using data, students continue to apply the process of mathematical modelling to analyse real-life situations.

Strand E: Geometry and Measurement

In this strand, students make connections among various geometric properties and their real-life applications. Students analyse and create designs to extend their understanding of geometric relationships to include circle and triangle properties. Students solve problems using different units within and between various measurement systems, examine the relationships between the volume of cones and cylinders and of pyramids and prisms, and solve problems that involve the application of perimeter, area, surface area, and volume.

Strand F: Financial Literacy

In this strand, students analyse financial situations to explain how mathematics can be used to understand such situations and inform financial decisions. They extend their financial literacy knowledge to answer questions related to appreciation and depreciation, and explain how budgets can be modified based on changes in circumstances. Students compare the effects of different interest rates, down payments, and other factors associated with purchasing goods and services. Students use their learning from other strands to solve financial problems of interest.

Some Considerations for Program Planning

Teachers consider many factors when planning a mathematics program that cultivates an inclusive environment in which all students can maximize their mathematical learning. This section highlights the key strategies and approaches that teachers and school leaders should consider as they plan effective and inclusive mathematics programs. Additional information can be found in the "Considerations for Program Planning" section, which provides information applicable to all curricula.

Instructional Approaches in Mathematics

Instruction in mathematics should support all students in acquiring the knowledge, skills, and habits of mind that they need in order to achieve the curriculum expectations and be able to enjoy and participate in mathematics learning for years to come.

Effective mathematics instruction begins with knowing the complex identities and profiles of the students, having high academic expectations for and of all students, providing supports when needed, and believing that all students can learn and do mathematics. Teachers incorporate Culturally Responsive and Relevant Pedagogy (CRRP) and provide authentic learning experiences to meet individual students' learning strengths and needs. Effective mathematics instruction focuses on the development of conceptual understanding and procedural fluency, skill development, and communication, as well as problem-solving skills. It takes place in a safe and inclusive learning environment, where all students are valued, empowered, engaged, and able to take risks, learn from mistakes, and approach the learning of mathematics in a confident manner. Instruction that is student centred and asset based builds effectively on students' strengths to develop mathematical habits of mind, such as curiosity and open-mindedness; a willingness to question, to challenge and be challenged; and an awareness of the value of listening intently, reading thoughtfully, and communicating with clarity.

Learning should be relevant: embedded in the lived realities of all students and inspired by authentic, real-life contexts as much as possible. This approach allows students to develop key mathematical concepts and skills, to appreciate the beauty and wide-ranging nature of mathematics, and to realize the potential of mathematics to raise awareness and effect social change that is innovative and sustainable. A focus on making learning relevant supports students in their use of mathematical reasoning to make connections throughout their lives.

Universal Design for Learning (UDL) and Differentiated Instruction (DI)

Students in every mathematics classroom vary in their identities, lived experiences, personal interests, learning profiles, and readiness to learn new concepts and skills. Universal Design for Learning (UDL) and differentiated instruction (DI) are robust and powerful approaches to designing assessment and instruction to engage all students in mathematical tasks that develop conceptual understanding and procedural fluency. Providing each student with opportunities to be challenged and to succeed requires teachers to attend to student differences and provide flexible and responsive approaches to instruction.

UDL and DI can be used in combination to help teachers respond effectively to the strengths and needs of all students.

The aim of the UDL framework is to assist teachers in designing mathematics programs and environments that provide all students with equitable access to the mathematics curriculum. Within this framework, teachers engage students in multiple ways in order to support them in becoming purposeful and motivated in their mathematics learning. Teachers take into account students’ diverse learner profiles by designing tasks that offer individual choice, ensuring relevance and authenticity, providing graduated levels of challenge, and fostering collaboration in the mathematics classroom. Teachers also represent concepts and information in multiple ways to help students become resourceful and knowledgeable learners. For example, teachers use a variety of media to ensure that students are provided with alternatives for auditory and visual information; they clarify mathematics vocabulary and symbols; and they highlight patterns and big ideas to guide information processing. To support learners as they focus strategically on their learning goals, teachers create an environment in which learners can express themselves using a range of kinesthetic, visual, and auditory strengths. For example, teachers can improve access to tools or assistive devices; vary ways in which students can respond and demonstrate their understanding of concepts; and support students in goal-setting, planning, and time-management skills related to their mathematics learning.

Designing mathematics tasks through UDL allows the learning to be “low floor, high ceiling” – that is, all students are provided with the opportunity to find their own entry point to the learning. Teachers can then support students in working at their own pace and provide further support as needed, while continuing to move student learning forward. Tasks that are intentionally designed to be low floor, high ceiling provide opportunities for students to use varied approaches and to continue to be engaged in learning with varied levels of complexity and challenge. This is an inclusive approach that is grounded in a growth mindset: the belief that everyone can do well in mathematics.

While UDL provides teachers with broad principles for planning mathematics instruction and learning experiences for a diverse group of students, DI allows them to address specific skills and learning needs. DI is rooted in assessment and involves purposefully planning varied approaches to teaching the content of the curriculum; to the processes (e.g., tasks and activities) that support students as they make sense of what they are learning; to the ways in which students demonstrate their learning and the outcomes they are expected to produce; and to the learning environment. DI is student centred and involves a strategic blend of whole-class, small-group, and individual learning activities to suit students’ differing strengths, interests, and levels of readiness to learn. Attending to students’ varied readiness for learning mathematics is an important aspect of differentiated teaching. Learners who are ready for greater challenges need support in aiming higher, developing belief in excellence, and co-creating problem-based tasks to increase the complexity while still maintaining joy in learning. Students who are struggling to learn a concept need to be provided with the scaffolding and encouragement to reach high standards. Through an asset-based approach, teachers focus on these learners’ strengths, imbuing instructional approaches with a strong conviction that all students can learn. To make certain concepts more accessible, teachers can employ strategies such as offering students choice, and providing open-ended problems that are based on relevant real-life situations and supported with visual and hands-on learning. Research indicates that using differentiated instruction in mathematics classrooms can diminish inequities.

Universal Design for Learning and differentiated instruction are integral aspects of an inclusive mathematics program and the achievement of equity in mathematics education. More information on these approaches can be found in the ministry publication *Learning for All: A Guide to Effective Assessment and Instruction for All Students, Kindergarten to Grade 12* (2013).

High-Impact Practices

Teachers understand the importance of knowing the identities and profiles of all students and of choosing the instructional approaches that will best support student learning. The approaches that teachers employ vary according to both the learning outcomes and the needs of the students, and teachers choose from and use a variety of accessible, equitable high-impact instructional practices.

The thoughtful use of these high-impact instructional practices – including knowing when to use them and how they might be combined to best support the achievement of specific math goals – is an essential component of effective math instruction. Researchers have found that the following practices consistently have a high impact on teaching and learning mathematics:[4]

- **Learning Goals, Success Criteria, and Descriptive Feedback.** Learning goals and success criteria outline the intention for the lesson and how this intention will be achieved to ensure teachers and students have a clear and common understanding of what is being learned and what success looks like. The use of descriptive feedback involves providing students with the precise information they need in order to reach the intended learning goal.
- **Direct Instruction.** This is a concise, intentional form of instruction that begins with a clear learning goal. It is not a lecture or a show-and-tell. Instead, direct instruction is a carefully planned and focused approach that uses questioning, activities, or brief demonstrations to guide learning, check for understanding, and make concepts clear. Direct instruction prioritizes feedback and formative assessment throughout the learning process and concludes with a clear summary of the learning that can be provided in written form, orally, and/or visually.
- **Problem-Solving Tasks and Experiences.** It is an effective practice to use a problem, intentionally selected or created by the teacher or students, to introduce, clarify, or apply a concept or skill. This practice provides opportunities for students to demonstrate their agency by representing, connecting, and justifying their thinking. Students communicate and reason with one another and generate ideas that the teacher connects in order to highlight important concepts, refine existing understanding, eliminate unsuitable strategies, and advance learning.
- **Teaching about Problem Solving.** Teaching students about the process of problem solving makes explicit the critical thinking that problem solving requires. It involves teaching students to identify what is known and unknown, to draw on similarities and differences between various types of problems, and to use representations to model the problem-solving situation.

[4] John Hattie, Douglas Fisher, Nancy Frey, Linda M. Gojak, Sara Delano Moore, and William Mellman, *Visible Learning for Mathematics: What Works Best to Optimize Student Learning, Grades K–12* (Thousand Oaks, CA: Corwin Mathematics, 2017).

- **Tools and Representations.** The use of a variety of appropriate tools and representations supports a conceptual understanding of mathematics. Carefully chosen and used effectively, representations and tools such as manipulatives make math concepts accessible to a wide range of learners. At the same time, student interactions with representations and tools also give teachers insight into students' thinking and learning.
- **Math Conversations.** Effective mathematical conversations create opportunities for all students to express their mathematical thoughts and to engage meaningfully in mathematical talk by listening to and responding to the ideas of others. These conversations involve reasoning, proving, building on the thinking of others, defending and justifying their own thinking, and adjusting their perspectives as they build their mathematical understanding, confidence, and awareness of the mathematical thoughts of others.
- **Small-Group Instruction.** A powerful strategy for moving student learning forward, small-group instruction involves targeted, timely, and scaffolded mathematics instruction that meets the learning needs of specific students at appropriate times. By working with small and flexible groups, whether they are homogenous or heterogenous, teachers can personalize learning in order to close gaps that exist or extend thinking. Small-group instruction also provides opportunities for teachers to connect with and learn more about student identities, experiences, and communities, which the teachers can build on as a basis for their mathematics instruction.
- **Deliberate Practice.** Practice is best when it is purposeful and spaced over time. It must always follow understanding and should be continual and consistent. Teachers provide students with timely descriptive feedback to ensure that students know they are practising correctly and sufficiently. Students also need to practise metacognition, or reflecting on their learning, in order to become self-directed learners.
- **Flexible Groupings.** The intentional combination of large-group, small-group, partnered, and independent working arrangements, in response to student and class learning needs, can foster a rich mathematical learning environment. Creating flexible groupings in a mathematics class enables students to work independently of the teacher but with the support of their peers, and it strengthens collaboration and communication skills. Regardless of the size of the group, it is of utmost importance that individual students have ownership of their learning.

While a lesson may prominently feature one of these high-impact practices, other practices will inevitably also be involved. The practices are rarely used in isolation, nor is there any single "best" instructional practice. Teachers strategically choose the right practice, for the right time, in order to create an optimal learning experience for all students. They use their socio-cultural awareness of themselves and their students, a deep understanding of the curriculum and of the mathematics that underpins the expectations, and a variety of assessment strategies to determine which high-impact instructional practice, or combination of practices, best supports the students. These decisions are made continually throughout a lesson. The appropriate use of high-impact practices plays an important role in supporting student learning.

More information can be found in the resource section on high-impact practices in mathematics.

When teachers effectively implement Universal Design for Learning, differentiated instruction, and high-impact practices in mathematics programs, they create opportunities for students to develop mathematics knowledge and skills, to apply mathematical processes, and to develop transferable skills that can be applied in other curricular areas.

The Role of Information and Communication Technology in Mathematics

The mathematics curriculum was developed with the understanding that the strategic use of technology is part of a balanced mathematics program. Technology can extend and enrich teachers' instructional strategies to support all students' learning in mathematics. Technology, when used in a thoughtful manner, can support and foster the development of mathematical reasoning, problem solving, and communication. For some students, technology is essential and required to access curriculum.

When using technology to support the teaching and learning of mathematics, teachers consider the issues of student safety, privacy, ethical responsibility, equity and inclusion, and well-being.

The strategic use of technology to support the achievement of the curriculum expectations requires a strong understanding of:

- the mathematical concepts being addressed;
- high-impact teaching practices that can be used, as appropriate, to achieve the learning goals;
- the capacity of the chosen technology to augment the learning, and how to use this technology effectively.

Technology (e.g., digital tools, computation devices, calculators, data-collection programs and coding environments) can be used specifically to support students' thinking in mathematics, to develop conceptual understanding (e.g., visualization using virtual graphing or geometry tools), and to facilitate access to information and allow better communication and collaboration (e.g., collaborative documents and web-based content that enable students to connect with experts and other students; language translation applications).

Coding has been introduced into the Grade 9 mathematics course as a continuum from the elementary mathematics curriculum. The elementary mathematics curriculum outlines a developmental progression for students to develop foundational coding skills. In Grade 9, students transition to using coding as a tool to interact with the mathematics they are learning. They use the skills developed in elementary to create and alter code in a multitude of coding environments including text-based programming languages, spreadsheets, computer algebra systems (CAS), and virtual graphing and geometry tools.

Technology can support English language learners in accessing mathematics terminology and ways of solving problems in their first language. Assistive technologies are critical in enabling some students with special education needs to have equitable access to the curriculum and in supporting their learning, and must be provided in accordance with a student's Individual Education Plan (IEP).

Technologies are important problem-solving tools. Computers and calculators are tools of mathematicians, and students should be given opportunities to select and use the learning tools that may be helpful or necessary for them as they search for their own solutions to problems.

Teachers understand the importance of technology and how it can be used to access and support learning for all students. Additional information can be found in the "The Role of Information and Communications Technology" subsection of "Considerations for Program Planning".

Education and Career/Life Planning

Education and career/life planning supports students in their transition from secondary school to their initial postsecondary destinations, whether in apprenticeships, college, community living, university, or the workplace.

Mathematics teachers can support students in education and career/life planning by making authentic connections between the mathematics concepts students are learning in school and the knowledge and skills needed in different careers. These connections engage students' interest and allow them to develop an understanding of the usefulness of mathematics in the daily lives of workers.

Teachers can promote students' awareness of careers involving mathematics by exploring real-life applications of mathematics concepts and providing opportunities for career-related project work. Such activities allow students to investigate mathematics-related careers compatible with their interests, aspirations, and abilities.

Community members can also act as a valuable resource by sharing their career expertise and supporting students in understanding the relevance of mathematics to various fields of study and careers. Career fairs, guest speakers, and job-shadowing days can provide opportunities for students to identify and explore mathematics-related careers.

Students may need support to comprehend the wide variety of professions and careers where mathematical concepts and processes are used. For example:

- fractions and imperial measures are used in various trades and daily activities;
- rates and percentages are used in banking, investing, and currency exchange;
- ratios and proportions are used in architecture, engineering, construction, nursing, pharmacy practice, hair colouring techniques, and fields related to culinary arts;
- algebraic reasoning is used in the sciences and computer programming;
- geometry and measurement concepts are used in construction, civil engineering, and art; statistics are used in real estate, the retail sector, tourism and recreation, conservation, finance, insurance, sports management, and research.

Students should be made aware that mathematical literacy, problem solving, and the other skills and knowledge they learn in the mathematics classroom are valuable assets in an ever-widening range of jobs and careers in today's society. More information can be found in the "Education and Career/Life Planning" subsection of "Considerations for Program Planning".

Planning Mathematics Programs for Students with Special Education Needs

Classroom teachers hold high expectations of all students and are the key educators in designing and supporting mathematics assessment and instruction for students with special education needs. They have a responsibility to support all students in their learning and to work collaboratively with special education teachers, where appropriate, to plan, design and implement appropriate instructional and assessment accommodations and modifications in the mathematics program to achieve this goal. More information on planning for and assessing students with special education needs can be found in the "Planning for Students with Special Education Needs" subsection of "Considerations for Program Planning".

Principles for Supporting Students with Special Education Needs

The following principles[5] guide teachers in effectively planning and teaching mathematics programs to students with special education needs, and also benefit all students:

- The teacher plays a critical role in student success in mathematics.
- It is important for teachers to develop an understanding of the general principles of how students learn mathematics.
- The learning expectations outline interconnected, developmentally appropriate key concepts and skills of mathematics across all of the strands.
- It is important to support students in making connections between procedural knowledge and conceptual understanding of mathematics.
- The use of concrete, visual, and virtual representations and tools is fundamental to learning mathematics and provides a way of representing both concepts and student understanding.
- The teaching and learning process involves ongoing assessment. Students with special education needs should be provided with various opportunities to demonstrate their learning and thinking in multiple ways.

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[5] Adapted from Ontario Ministry of Education, *Education for All: The Report of the Expert Panel on Literacy and Numeracy Instruction for Students with Special Education Needs, Kindergarten to Grade 6* (Toronto, ON: Author, 2005).

An effective mathematics learning environment and program that addresses the mathematical learning needs of students with special education needs is purposefully planned with the principles of Universal Design for Learning in mind and integrates the following elements:

- knowing the student's cultural and linguistic background, strengths, interests, motivations, and needs in mathematics learning in order to differentiate learning and make accommodations and modifications as outlined in the student's Individual Education Plan;
- building the student's confidence and positive identity as a mathematics learner;
- valuing the student's prior knowledge and connecting what the student knows with what the student needs to learn;
- identifying and focusing on the connections between broad concepts in mathematics;
- connecting mathematics with familiar, relevant, everyday situations and providing rich and meaningful learning contexts;
- fostering a positive attitude towards mathematics and an appreciation of mathematics through multimodal means, including through the use of assistive technology and the performance of authentic tasks;
- implementing research-informed instructional approaches (e.g., Concrete – Semi-Concrete – Representational – Abstract) when introducing new concepts to promote conceptual understanding, procedural accuracy, and fluency;
- creating a balance of explicit instruction, problem solving within a student's zone of proximal development, learning in flexible groupings, and independent learning. Each instructional strategy should take place in a safe, supportive, and stimulating environment while taking into consideration that some students may require more systematic and intensive support, and more explicit and direct instruction, before engaging in independent learning;
- assessing student learning through observations, conversations with the students, and frequent use of low-stakes assessment check-ins and tools;
- providing immediate feedback in order to facilitate purposeful, correct practice that supports understanding of concepts and procedures, as well as efficient strategies;
- providing environmental, assessment, and instructional accommodations in order to maximize the student's learning (e.g., making available learning tools such as virtual manipulatives, computer algebra systems, and calculators; ensuring access to assistive technology), as well as modifications that are specified in the student's Individual Education Plan;
- building an inclusive community of learners and encouraging students with special education needs to participate in various mathematics-oriented class projects and activities;
- building partnerships with administrators and other teachers, particularly special education teachers, where available, to share expertise and knowledge of the curriculum expectations; co-develop content in the Individual Education Plan that is specific to mathematics; and systematically implement intervention strategies, as required, while making meaningful connections between school and home to ensure that what the student is learning in the school is relevant and can be practised and reinforced beyond the classroom.

Planning Mathematics Programs for English Language Learners

English language learners are working to achieve the curriculum expectations in mathematics while they are developing English-language proficiency. An effective mathematics program that supports the success of English language learners is purposefully planned with the following considerations in mind.

- Students' various linguistic identities are viewed as a critical resource in mathematics instruction and learning. Recognizing students' language resources and expanding linguistic competence enables students to use their linguistic repertoire in a fluid and dynamic way, mixing and meshing languages to communicate. This translingual practice[6] is creative and strategic, and allows students to communicate, interact, and connect with peers and teachers using the full range of their linguistic repertoire, selecting features and modes that are most appropriate to communicate across a variety of purposes, such as when developing conceptual knowledge and when seeking clarity and understanding.
- Students may be negotiating between school-based mathematics and ways of mathematical reasoning from diverse cultural and linguistic backgrounds. They may have deep mathematical knowledge and skills developed in another educational cultural and/or linguistic context, and may already have learned the same mathematical terms and concepts that they are studying now, but in another language.
- Knowledge of the diversity among English language learners and of their mathematical strengths, interests, and identities, including their social and cultural backgrounds, is important. These "funds of knowledge"[7] are historically and culturally developed skills and assets that can be incorporated into mathematics learning to create a richer and more highly scaffolded learning experience for all students, promoting a positive, inclusive teaching and learning environment. Understanding how mathematical concepts are described in students' home languages and cultures can provide insight into how students are thinking about mathematical ideas.[8]
- In addition to assessing their level of English-language proficiency, an initial assessment of the mathematics knowledge and skills of newcomer English language learners is required in Ontario schools.

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[6] Ofelia García, Susana Ibarra Johnson, and Kate Seltzer, *The Translanguaging Classroom: Leveraging Student Bilingualism for Learning* (Philadelphia, PA: Caslon, 2017).Sunny Man Chu Lau and Saskia Van Viegen, *Plurilingual Pedagogies* (Springer Cham, New York City, NY: 2020).

[7] Elizabeth Marshall and Kelleen Toohey, "Representing Family: Community Funds of Knowledge, Bilingualism, and Multimodality," *Harvard Educational Review* 80, no. 2 (2010), 221–42.

[8] Lisa Lunney Borden, "What's the Word for…? Is There a Word for…? How Understanding Mi'kmaw Language Can Help Support Mi'kmaw Learners in Mathematics", *Mathematics Education Research Journal* 25, no. 1 (2013): 5–22.

- Differentiated instruction is essential in supporting English language learners, who face the dual challenge of learning new conceptual knowledge while acquiring English-language proficiency. Designing mathematics learning to have the right balance for English language learners is achieved through program adaptations (e.g., accommodations that utilize their background knowledge in their first language) that ensure the tasks are mathematically challenging, reflective of learning demands within the mathematics curriculum, and comprehensible and accessible to English language learners. Using the full range of a student's language assets, including additional languages that a student speaks, reads, and writes, as a resource in the mathematics classroom supports access to their prior learning, reduces the language demands of the mathematics curriculum, and increases engagement.
- Working with students and their families and with available community supports allows for the multilingual representation of mathematics concepts to create relevant and real-life learning contexts and tasks.

In a supportive learning environment, scaffolding the learning of mathematics assessment and instruction offers English language learners the opportunity to:

- integrate their linguistic repertoire rather than engage in language separation, and select and use the linguistic features and modes that are most appropriate for their communication purposes;
- discuss how mathematical concepts are described in their language(s) and cultures;[9]
- draw on their additional language(s) (e.g., some newcomer students may use technology to access mathematical terminology and ways of solving problems in their first language), prior learning experiences, and background knowledge in mathematics;
- learn new mathematical concepts in authentic, meaningful, and familiar contexts;
- engage in open and parallel tasks to allow for multiple entry points for learning;
- work in a variety of settings that support co-learning and multiple opportunities for practice (e.g., with partners or in small groups with same-language peers, as part of cooperative or collaborative learning, in group conferences);
- access the language of instruction during oral, written, and multimodal instruction and assessment, during questioning, and when encountering texts, learning tasks, and other activities in mathematics;
- use oral language in different strategically planned activities, such as "think-pair-share", "turn-and-talk", and "adding on", to express their ideas and engage in mathematical discourse;
- develop both everyday and academic vocabulary, including specialized mathematics vocabulary in context, through rephrasing and recasting by the teacher and through using student-developed bilingual word banks or glossaries;
- practise using sentence frames adapted to their English-language proficiency levels to describe concepts, provide reasoning, hypothesize, make judgements, and explain their thinking;

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[9] Lisa Lunney Borden, "What's the Word for…? Is There a Word for…? How Understanding Mi'kmaw Language Can Help Support Mi'kmaw Learners in Mathematics", *Mathematics Education Research Journal* 25, no. 1 (2013): 5–22.

- use a variety of concrete and/or digital learning tools to demonstrate their learning in mathematics in multiple ways (e.g., orally, visually, kinesthetically), through a range of representations (e.g., portfolios, displays, discussions, models), and in multiple languages (e.g., multilingual word walls and anchor charts);
- have their learning assessed in terms of the processes they use in multiple languages, both during the learning and through teachers' observations and conversations.

Strategies used to differentiate instruction and assessment for English language learners in the mathematics classroom also benefit many other learners in the classroom, since programming is focused on leveraging all students' strengths, meeting learners where they are in their learning, being aware of language demands in mathematics, and making learning visible. For example, different cultural approaches to solve mathematical problems can help students make connections to the Ontario curriculum and provide classmates with alternative ways of solving problems.

English language learners in English Literacy Development (ELD) programs require accelerated support to develop both their literacy skills and their numeracy skills. These students have significant gaps in their education because of limited or interrupted opportunities for or access to schooling. In order to build a solid foundation of mathematics, they are learning key mathematical concepts missed in prior years. At the same time, they are learning the academic language of mathematics in English while not having acquired developmentally appropriate literacy skills in their first language. Programming for these students is therefore highly differentiated and intensive. These students often require focused support over a longer period than students in English as a Second Language (ESL) programs. The use of students' oral competence in languages other than English is a non-negotiable scaffold. The strategies described above, such as the use of visuals, the development of everyday and academic vocabulary, the use of technology, and the use of oral competence, are essential in supporting student success in ELD programs and in mathematics.

Supporting English language learners is a shared responsibility. Collaboration with administrators and other teachers, particularly ESL/ELD teachers, and Indigenous representatives, where possible, contributes to creating equitable outcomes for English language learners. Additional information on planning for and assessing English language learners can be found in the "Planning for English Language Learners" subsection of "Considerations for Program Planning".

Cross-Curricular and Integrated Learning in Mathematics

When planning an integrated mathematics program, educators should consider that, although the mathematical content in the curriculum is outlined in discrete strands, students develop mathematical thinking, such as proportional reasoning, algebraic reasoning, and spatial reasoning, that transcends the expectations in the strands and even connects to learning in other subject areas. By purposefully drawing connections across all areas of mathematics and other subject areas, and by applying learning to relevant real-life contexts, teachers extend and enhance student learning experiences and deepen their knowledge and skills across disciplines and beyond the classroom.

For example, proportional reasoning, which is developed through the study of ratios and rates in the Number strand, is also used when students are working towards meeting learning expectations in other strands of the math curriculum, such as in Geometry and Measurement and in Algebra, and in other disciplines, such as science, geography, and the arts. Students then apply this learning in their everyday lives – for example, when adjusting a recipe, preparing a mixture or solutions, or making unit conversions.

Similarly, algebraic reasoning is applied beyond the Number and Algebra strands. For example, it is applied in measurement when learning about formulas, such as volume of a pyramid $= \frac{\text{area of the base} \times \text{height}}{3}$. It is applied in other disciplines, such as science, when students study simple machines and learn about the formula *work* = *force* × *distance*. Algebraic reasoning is also used when making decisions in everyday life – for example, when determining which service provider offers a better consumer contract or when calculating how much time it will take for a frozen package to thaw.

Spatial thinking has a fundamental role throughout the Ontario curriculum, from Kindergarten to Grade 12, including in mathematics, the arts, health and physical education, and science. For example, a student demonstrates spatial reasoning when mentally rotating and matching shapes in mathematics, navigating movement through space and time, and using diagonal converging lines to create perspective drawings in visual art and to design and construct objects. In everyday life, there are many applications of spatial reasoning, such as when creating a garden layout or when using a map to navigate the most efficient way of getting from point A to point B.

Algebraic and proportional reasoning and spatial thinking are integral to all STEM disciplines. For example, students may apply problem-solving skills and mathematical modelling through engineering design as they build and test a prototype and design solutions intended to solve complex real-life problems. Consider how skills and understanding that students gain across the strands of the Grade 9 Mathematics course, such as Financial Literacy, Number, and Data, can be integrated into real-life activities. For example, as students collect financial data relating to compound interest, and examine patterns in the data involving compound interest, they apply their understanding of exponents and non-linear growth to generalize rules that can be coded in technology programming environments. This process allows students to create a variety of mathematical models and analyse them quantitatively. These models can then be used to support discussions about what factors can enable or constrain financial decision making, while taking ethical, societal, environmental, and personal considerations into account.

Teaching mathematics as a narrowly defined subject area places limits on the depth of learning that can occur. When teachers work together to develop integrated learning opportunities and highlight cross-curricular connections, students are better able to:

- make connections among the strands of the mathematics curriculum, and between mathematics and other subject areas;
- improve their ability to consider different strategies to solve a problem;
- debate, test, and evaluate whether strategies are effective and efficient;

- apply a range of knowledge and skills to solve problems in mathematics and in their daily experiences and lives.

When students are provided with opportunities to learn mathematics through real-life applications, integrating learning expectations from across the curriculum, they use their lived experiences and knowledge of other subject matter to enhance their learning of and engagement in mathematics. More information can be found in "Cross-Curricular and Integrated Learning".

Literacy in Mathematics

Literacy skills needed for reading and writing in general are essential for the learning of mathematics. To engage in mathematical activities and develop computational fluency, students require the ability to read and write mathematical expressions, to use a variety of literacy strategies to comprehend mathematical text, to use language to analyse, summarize, and record their observations, and to explain their reasoning when solving problems. Mathematical expressions and other mathematical texts are complex and contain a higher density of information than any other text.[10] Reading mathematical text requires literacy strategies that are unique to mathematics.

The learning of mathematics requires students to navigate discipline-specific reading and writing skills; therefore, it is important that mathematics instruction link literacy practices to specific mathematical processes and tasks. To make their thinking visible, students should be encouraged to clearly communicate their mathematical thinking, using the discipline-specific language of mathematics, which provides educators with the opportunity to correct student thinking when necessary.[11] The language of mathematics includes special terminology. To support all students in developing an understanding of mathematical texts, teachers need to explicitly teach mathematical vocabulary, focusing on the many meanings and applications of the terms students may encounter. In mathematics, students are required to use appropriate and correct terminology and are encouraged to use language with care and precision in order to communicate effectively.

More information about the importance of literacy across the curriculum can be found in the "Literacy" and "Mathematical Literacy" subsections of "Cross-curricular and Integrated Learning".

Transferable Skills in Mathematics

The Ontario curriculum emphasizes a set of skills that are critical to all students' ability to thrive in school, in the world beyond school, and in the future. These are known as transferable skills. Educators

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[10] Joan M. Kenney et al., *Literacy Strategies for Improving Mathematics Instruction* (Alexandria, VA: Association for Supervision and Curriculum Development, 2005).

[11] William G. Brozo and Sarah Crain, "Writing in Math: A Disciplinary Literacy Approach", *The Clearing House: A Journal of Educational Strategies, Issues and Ideas* 91, no. 7 (2017): 2.

facilitate students’ development of transferable skills across the curriculum, from Kindergarten to Grade 12. They are as follows:

- **Critical Thinking and Problem Solving.** In mathematics, students and educators learn and apply strategies to understand and solve problems flexibly, accurately, and efficiently. They learn to understand and visualize a situation and to use the tools and language of mathematics to reason, make connections to real-life situations, communicate, and justify solutions.
- **Innovation, Creativity, and Entrepreneurship.** In mathematics, students and educators solve problems with curiosity, creativity, and a willingness to take risks. They pose questions, make and test conjectures, and consider problems from different perspectives to generate new learning and apply it to novel situations.
- **Self-Directed Learning.** By reflecting on their own thinking and emotions, students, with the support of educators, can develop perseverance, resourcefulness, resilience, and a sense of self. In mathematics, they initiate new learning, monitor their thinking and their emotions when solving problems, and apply strategies to overcome challenges. They perceive mathematics as useful, interesting, and doable, and confidently look for ways to apply their learning.
- **Collaboration.** In mathematics, students and educators engage with others productively, respectfully, and critically in order to better understand ideas and problems, generate solutions, and refine their thinking.
- **Communication.** In mathematics, students and educators use the tools and language of mathematics to describe their thinking and to understand the world. They use mathematical vocabulary, symbols, conventions, and representations to make meaning, express a point of view, and make convincing and compelling arguments in a variety of ways, including multimodally; for example, using combinations of oral, visual, textual, and gestural communication.
- **Global Citizenship and Sustainability.** In mathematics, students and educators recognize and appreciate multiple ways of knowing, doing, and learning, and value different perspectives. They recognize how mathematics is used in all walks of life and how engaged citizens can use it as a tool to raise awareness and generate solutions for various political, environmental, social, and economic issues.
- **Digital Literacy.** In mathematics, students and educators learn to be discerning users of technology. They select when and how to use tools to understand and model real-life situations, predict outcomes, and solve problems, and they assess and evaluate the reasonableness of their results.

Transferable skills can be developed through the effective implementation of high-impact instructional strategies. More information can be found in “Transferable Skills”.

Assessment and Evaluation of Student Achievement

Growing Success: Assessment, Evaluation, and Reporting in Ontario Schools, First Edition, Covering Grades 1 to 12, 2010 sets out the Ministry of Education's assessment, evaluation, and reporting policy. The policy aims to maintain high standards, improve student learning, and benefit all students, parents, and teachers in elementary and secondary schools across the province. Successful implementation of this policy depends on the professional judgement[12] of teachers at all levels as well as their high expectations of all students, and on their ability to work together and to build trust and confidence among parents and students.

Major aspects of assessment, evaluation, and reporting policy are summarized in the main "Assessment and Evaluation" section. The key tool for assessment and evaluation in mathematics – the achievement chart – is provided below.

Culturally Responsive and Relevant Assessment and Evaluation in Mathematics

Culturally Responsive and Relevant Pedagogy (CRRP) reflects and affirms students' racial and social identities, languages, and family structures. It involves careful acknowledgement, respect, and understanding of the similarities and differences among students, and between students and teachers, in order to respond effectively to student thinking and promote student learning.

Engaging in assessment from a CRRP stance requires that teachers gain awareness of and interrogate their own beliefs about who a mathematical learner is and what they can achieve (see the questions for consideration provided below). In this process, teachers engage in continual self-reflection – and the critical analysis of various data – to understand and address the ways in which power and privilege affect the assessment and evaluation of student learning. Assessment from a CRRP stance starts with having a deep knowledge of every student and understanding of how they learn best. Teachers seek to build authentic, trusting relationships with students, and with their families and community, as they seek opportunities to build new understanding and support equitable outcomes for all students.

Assessment from a CRRP stance, by its nature, encompasses a wide variety of assessment approaches. It is designed to reflect, affirm, and enhance the multiple ways of knowing and being that students bring to the classroom while maintaining appropriate and high academic expectations for all students. The primary purpose of assessment is to improve student learning. Assessment *for* learning creates opportunities for teachers to intentionally learn about each student and their sociocultural and linguistic background in order to gather a variety of evidence about their learning in an anti-racist, anti-discriminatory environment, in a way that is reflective of and responsive to each student's strengths, experiences, interests, and cultural ways of knowing. Ongoing descriptive feedback and responsive coaching for improvement is essential for improving student learning.

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[12] "Professional judgement", as defined in *Growing Success* (p. 152), is "judgement that is informed by professional knowledge of curriculum expectations, context, evidence of learning, methods of instruction and assessment, and the criteria and standards that indicate success in student learning. In professional practice, judgement involves a purposeful and systematic thinking process that evolves in terms of accuracy and insight with ongoing reflection and self-correction".

Teachers engage in assessment *as* learning by creating ongoing opportunities for all students to develop their capacity to be confident, independent, autonomous learners who set individual goals, monitor their own progress, determine next steps, and reflect on their thinking and learning in relation to learning goals and curriculum expectations. Teachers engage in culturally responsive and relevant practices by supporting students in the development of these skills by holding positive and affirming views of their students and of their ability to learn and achieve academic success. One way in which teachers differentiate assessment is by providing tasks that allow multiple entry points for all students to engage and that enable all students to access complex mathematics.

Assessment *of* learning is used by the teacher to summarize learning at a given point in time. This summary is used to make judgements about the quality of student learning on the basis of established criteria, to assign a value to represent that quality, and to support the communication of information about achievement to each student, parents, teachers, and others. Teachers engage in culturally responsive and relevant practices that honour and value the importance of student agency and voice in determining the variety of ways in which students can demonstrate their learning.

The evidence that is collected about student learning, including observations and conversations as well as student products, should reflect and affirm the student's lived experiences within their school, home, and community, learning strengths, and mathematical knowledge. This process of triangulating evidence of student learning allows teachers to improve the accuracy of their understanding with respect to how each student is progressing in their learning. Assessment that is rooted in CRRP is an equitable, inclusive, and transparent process that values students as active participants in their learning.

When teachers engage in the process of examining their own biases regarding classroom assessment and evaluation practices, they might consider some of the following questions:

- Are the tasks accessible to, and inclusive of, all learners? Do the tasks include appropriate and varied entry points for all students?
- Do the tasks connect to students' prior learning and give them opportunities to be sense makers and to integrate their new learning? Do the selected tasks reflect students' identities and lived experiences?
- Do all students have equitable access to the tools they need to complete the tasks being set?
- What opportunities can teachers build into their practice to offer students descriptive feedback to enhance learning? Are graded assessment tasks used in a way that complements the use of descriptive feedback for growth?
- How can information be conveyed about students' learning progress to students and parents in an ongoing and meaningful way?
- What is the purpose of assigning and grading a specific task or activity? Are student choice and agency considered?
- How do teacher biases influence decisions about what tasks or activities are chosen for assessment?

The Achievement Chart for Grade 9 Mathematics

The achievement chart identifies four categories of knowledge and skills and four levels of achievement in mathematics. (For important background, see “Content Standards and Performance Standards” in the main Assessment and Evaluation section.)

Knowledge and Understanding – Subject-specific content acquired in each grade (knowledge), and the comprehension of its meaning and significance (understanding)				
Categories	**50–59% (Level 1)**	**60–69% (Level 2)**	**70–79% (Level 3)**	**80–100% (Level 4)**
	The student:			
Knowledge of content *(e.g., terminology, procedural skills, mathematical models)*	demonstrates limited knowledge of content	demonstrates some knowledge of content	demonstrates considerable knowledge of content	demonstrates thorough knowledge of content
Understanding of content *(e.g., concepts, principles, mathematical structures and processes)*	demonstrates limited understanding of content	demonstrates some understanding of content	demonstrates considerable understanding of content	demonstrates thorough understanding of content
Thinking – The use of critical and creative thinking skills and/or processes				
Categories	**50–59% (Level 1)**	**60–69% (Level 2)**	**70–79% (Level 3)**	**80–100% (Level 4)**
	The student:			
Use of planning skills *(e.g., understanding the problem; generating ideas; formulating a plan of action; selecting strategies, models, and tools; making conjectures and hypotheses)*	uses planning skills with limited effectiveness	uses planning skills with some effectiveness	uses planning skills with considerable effectiveness	uses planning skills with a high degree of effectiveness
Use of processing skills* *(e.g., carrying out a plan: collecting data, questioning, testing, revising, modelling, solving, inferring, forming conclusions; looking back at a solution: evaluating reasonableness, making arguments in support of a solution, reasoning, justifying, proving, reflecting)*	uses processing skills with limited effectiveness	uses processing skills with some effectiveness	uses processing skills with considerable effectiveness	uses processing skills with a high degree of effectiveness

Use of critical/creative thinking processes* *(e.g., posing and solving problems, critiquing solutions, using mathematical reasoning, evaluating mathematical models, making inferences and testing conjectures and hypotheses)*	uses critical/ creative thinking processes with limited effectiveness	uses critical/ creative thinking processes with some effectiveness	uses critical/ creative thinking processes with considerable effectiveness	uses critical/ creative thinking processes with a high degree of effectiveness
Communication – The conveying of meaning through various forms				
Categories	**50–59% (Level 1)**	**60–69% (Level 2)**	**70–79% (Level 3)**	**80–100% (Level 4)**
	The student:			
Expression and organization of ideas and information in oral, visual, and/or written forms *(e.g., pictorial, graphic, dynamic, numeric, algebraic forms; gestures and other non-verbal forms; models)*	expresses and organizes ideas and information with limited effectiveness	expresses and organizes ideas and information with some effectiveness	expresses and organizes ideas and information with considerable effectiveness	expresses and organizes ideas and information with a high degree of effectiveness
Communication for different audiences and purposes *(e.g., to share mathematical thinking, to inform, to persuade, to share findings)* **in oral, visual, and/or written forms**	communicates for different audiences and purposes with limited effectiveness	communicates for different audiences and purposes with some effectiveness	communicates for different audiences and purposes with considerable effectiveness	communicates for different audiences and purposes with a high degree of effectiveness
Use of conventions, vocabulary, and terminology of the discipline in oral, visual, and/or written forms *(e.g., terms, symbols, units, labels, structures)*	uses conventions, vocabulary, and terminology with limited effectiveness	uses conventions, vocabulary, and terminology with some effectiveness	uses conventions, vocabulary, and terminology with considerable effectiveness	uses conventions, vocabulary, and terminology with a high degree of effectiveness
Application – The use of knowledge and skills to make connections within and between various contexts				
Categories	**50–59% (Level 1)**	**60–69% (Level 2)**	**70–79% (Level 3)**	**80–100% (Level 4)**
	The student:			

Application of knowledge and skills *(e.g., selecting and using representations, mathematical tools, and strategies)* **in familiar contexts**	applies knowledge and skills in familiar contexts with limited effectiveness	applies knowledge and skills in familiar contexts with some effectiveness	applies knowledge and skills in familiar contexts with considerable effectiveness	applies knowledge and skills in familiar contexts with a high degree of effectiveness
Transfer of knowledge and skills *(e.g., selecting and using representations, mathematical tools, and strategies)* **to new contexts**	transfers knowledge and skills to new contexts with limited effectiveness	transfers knowledge and skills to new contexts with some effectiveness	transfers knowledge and skills to new contexts with considerable effectiveness	transfers knowledge and skills to new contexts with a high degree of effectiveness
Making connections within and between various contexts (*e.g., connections to real-life situations and lived experiences; connections among concepts and representations; connections between mathematics and other disciplines, including other STEM [science, technology, engineering, and mathematics] subjects)*	makes connections within and between various contexts with limited effectiveness	makes connections within and between various contexts with some effectiveness	makes connections within and between various contexts with considerable effectiveness	makes connections within and between various contexts with a high degree of effectiveness

* **Note**: The processing skills and critical/creative thinking processes in the Thinking category include some but not all aspects of the *mathematical processes* laid out in Strand A: Mathematical Thinking and Making Connections.

Requirements for Strand AA and Strand A

Strand AA: Social-Emotional Learning (SEL) Skills in Mathematics. Learning related to the expectation in Strand AA occurs in the context of learning related to the other six strands. The focus is on intentional instruction; learning in this strand is not included in the assessment, evaluation, or reporting of student achievement.

Strand A: Mathematical Thinking and Making Connections. Strand A has no specific expectations. Students' learning related to this strand takes place in the context of learning related to strands B through F. Student achievement of the expectations in Strand A is to be assessed and evaluated throughout the course.

Criteria and Descriptors for Grade 9 Mathematics

To guide teachers in their assessment and evaluation of student learning, the achievement chart provides “criteria” and “descriptors” within each of the four categories of knowledge and skills.

A set of criteria is identified for each category in the achievement chart. The criteria are subsets of the knowledge and skills that define the category. The criteria identify the aspects of student performance that are assessed and/or evaluated, and they serve as a guide to what teachers look for. In the mathematics curriculum, the criteria for each category are as follows:

Knowledge and Understanding

- knowledge of content (e.g., terminology, procedural skills, mathematical models)
- understanding of content (e.g., concepts, principles, mathematical structures and processes)

Thinking

- use of planning skills (e.g., understanding the problem; generating ideas; formulating a plan of action; selecting strategies, models, and tools; making conjectures and hypotheses)
- use of processing skills (e.g., carrying out a plan: collecting data, questioning, testing, revising, modelling, solving, inferring, forming conclusions; looking back at a solution: evaluating reasonableness, making arguments in support of a solution, reasoning, justifying, proving, reflecting)
- use of critical/creative thinking processes (e.g., posing and solving problems, critiquing solutions, using mathematical reasoning, evaluating mathematical models, making inferences and testing conjectures and hypotheses)

Communication

- expression and organization of ideas and information in oral, visual, and/or written forms (e.g., pictorial, graphic, dynamic, numeric, algebraic forms; gestures and other non-verbal forms; models)
- communication for different audiences and purposes (e.g., to share mathematical thinking, to inform, to persuade, to share findings) in oral, visual, and/or written forms
- use of conventions, vocabulary, and terminology of the discipline in oral, visual, and/or written forms (e.g., terms, symbols, units, labels, structures)

Application

- application of knowledge and skills (e.g., selecting and using representations, mathematical tools, and strategies) in familiar contexts
- transfer of knowledge and skills (e.g., selecting and using representations, mathematical tools, and strategies) to new contexts
- making connections within and between various contexts (e.g., connections to real-life situations and lived experiences; connections among concepts and representations; connections between mathematics and other disciplines, including other STEM [science, technology, engineering, and mathematics] subjects)

“Descriptors” indicate the characteristics of the student’s performance, with respect to a particular criterion, on which assessment or evaluation is focused. *Effectiveness* is the descriptor used for each of the criteria in the Thinking, Communication, and Application categories. What constitutes effectiveness in any given performance task will vary with the particular criterion being considered. Assessment of effectiveness may therefore focus on a quality such as appropriateness, clarity, accuracy, precision, logic, relevance, significance, fluency, flexibility, depth, or breadth, as appropriate for the particular criterion.

Expectations by Strand

Note

Teacher Supports

The expectations are accompanied by “teacher supports”, which may include examples, key concepts, teacher prompts, instructional tips, and/or sample tasks. These elements are intended to promote understanding of the intent of the specific expectations and are offered as illustrations for teachers. *The teacher supports do not set out requirements for student learning; they are optional, not mandatory*.

“Examples” are meant to illustrate the intent of the expectation, the kind of knowledge, concepts, or skills, the specific area of learning, the depth of learning, and/or the level of complexity that the expectation entails.

“Teacher prompts” are sample guiding questions and considerations that can lead to discussions and promote deeper understanding.

“Instructional tips” are intended to support educators in delivering instruction that facilitates student learning related to the knowledge, concepts, and skills set out in the expectations.

“Sample tasks” are developed to model appropriate practice for the course. They provide possible learning activities for teachers to use with students and illustrate connections between the mathematical knowledge, concepts, and skills. Teachers can choose to draw on the sample tasks that are appropriate for their classrooms, or they may develop their own approaches that reflect a similar level of complexity and high-quality mathematical instruction. Whatever the specific ways in which the requirements outlined in the expectations are implemented in the classroom, they must, wherever possible, be inclusive and reflect the diversity of the student population and the population of the province. When designing inclusive learning tasks, teachers reflect on their own biases and incorporate their deep knowledge of the curriculum, as well as their understanding of the diverse backgrounds, lived experiences, and identities of students. Teachers will notice that some of the sample tasks address the requirements of the expectation they are associated with and incorporate mathematical knowledge, concepts, or skills described in expectations in other strands of the course. Some tasks are cross-curricular in nature and will cover expectations in other disciplines in conjunction with the mathematics expectations.

AA. Social-Emotional Learning (SEL) Skills in Mathematics

Overall expectations

Throughout this course, in the context of learning related to the other strands, students will:

AA1. Social-Emotional Learning Skills

develop and explore a variety of social-emotional learning skills in a context that supports and reflects this learning in connection with the expectations across all other strands

This overall expectation is to be included in classroom instruction, but not in assessment, evaluation, or reporting. See further information about approaches to instruction that support all students as they work to apply mathematical thinking, make connections, and develop a healthy identity as mathematics learners to foster well-being and the ability to learn mathematics.

Teacher supports

Examples

The following examples illustrate various ways teachers may provide instruction to support students in developing social-emotional learning skills in connection to learning mathematics.

Recognizing and Identifying Emotions That Support Mathematical Learning

Scenario [checking observations with student]: A student has been given the task of multiplying powers with variable bases (e.g., $a^2 \cdot a^8$).

Teacher Observation: The teacher perceives that the student appears to be frustrated with the task.

Note: Each teacher's perspectives are subject to and informed by their own experiences. What the teacher observes and perceives may or may not align with what the student is actually feeling and experiencing. A culturally responsive and relevant approach starts with the teacher engaging in self-reflection, then considering elements of an inclusive learning environment and the educational context in which they are observing the student.

Teacher Action and Student Response: The teacher asks the student if they would like to talk through the task together. The student accepts. During the conversation, the teacher supports the student in identifying the emotions they are feeling and helps build the student's understanding that thoughts, feelings, and actions are all connected and affect one another. In this case, the student identifies that they are feeling confused and frustrated. The teacher works with the student to identify strategies that will help in this situation (e.g., identify what they do understand, develop related mathematical literacy, make connections by relating the task to what they know about powers with numeric bases, seek further information by reviewing class notes about exponent laws, explore different ways of looking at the problem, take a break and come back to the task later). In this case, the teacher asks the student to think about a similar situation and strategies they may have used when they multiplied powers with bases that were integers. For example, the student could be encouraged to write out the expanded form of the expression, without exponents, use that expression to recall what to do and why the method makes sense, then generalize this understanding and apply the method to the problem with the variable bases. In this way, the student makes the connection to their prior learning and applies it to complete the task. The student could then reflect on how the approach they took to work through this task might help them the next time they feel frustrated with a task.

Teacher Reflection [continue reflection on an ongoing basis]: The teacher reflects on how their actions may have affected the student's level of confidence in using strategies for continuing to problem solve when the student feels frustrated. The teacher thinks about potential alternatives for future interactions with this student. This interaction also supports teacher decisions on future tasks that may improve this particular student's perseverance skills and confidence.

Note: Ongoing teacher reflection is important throughout instruction, not just at the beginning and end. It includes developing an understanding of individual student identities, strengths, and needs, including language learning and educational experience. This is a critical first step in building trust and relationships with students.

Recognizing Sources of Stress That Present Challenges to Mathematical Learning

Scenario [using culturally responsive pedagogy to build a plan of action in response to individual student strengths and needs]: A student has been given a task that involves fractions.

Student's Initial Reaction: The student shares with the teacher that they get stressed every time they encounter a fraction.

Teacher and Student Conversation: The teacher thanks the student for sharing information about how they are feeling and asks if they would like to share more about why fractions create a stressful response for them. The student shares with the teacher that they do not see the relevance of fractions to their everyday life, and that they have always struggled with fractions for as long as they can remember.

The teacher acknowledges the student's feelings of stress, and helps the student respond to these feelings by identifying what they do know about fractions. Next steps could include identifying what additional support is needed to help the student feel less stressed and be more successful and confident when working with fractions. For example, the teacher could select mathematical tasks that are contextualized to provide a relatable entry point for this student, including the use of fraction manipulatives (e.g., fraction strips, relational rods) to complete the tasks.

Teacher Reflection [consider strengths and needs of individual students]: The teacher thinks about how their approach to working with individual students to identify personal strategies to respond to stress seemed to have been received by this particular student. This provides useful information about strategies that might best support this student in the future.

Identifying Resources and Supports That Aid Perseverance in Mathematical Learning

Scenario [providing one-on-one support]: A student has been given the task of writing code to explore what happens to the volume of a rectangular prism when one, two, and three dimensions of the prism are altered.

Student's Initial Reaction: The student shares with the teacher that they do not know how to begin the task and they feel overwhelmed.

Teacher's Response to Student: The teacher has a conversation with the student to learn about their prior experience with coding. The teacher then works directly with the student, supporting them as they develop a physical model and a flow chart to plan out the code for exploring what happens to the volume of a rectangular prism when one dimension changes. Next, the teacher has the student write the code and execute it to see if they get the output they were expecting.

Student's Response to Teacher: The student shares that the code they wrote produced what they expected.

Teacher's Response to Student: The teacher then works with the student to identify how the model and the flow chart will need to be altered to reflect a change in two dimensions instead of one. The teacher points out that knowing the steps to follow and knowing that it is okay if the results aren't as expected both aid in persevering with the task.

Student's Response to Teacher: The student shares with the teacher that they now know what to do and can proceed with writing the remaining code to complete the task.

After getting to this stage, the student reflects that one of the benefits of coding is that one can get feedback right away, and that it is okay if the results are not as expected. Identifying the reason for the difference is part of the process and the challenge.

Teacher Reflection [consider strategies for uplifting students and impact of this]: The teacher thinks about aspects of what this student did that could be helpful to other students. After inquiring whether this student is willing to share their successes, the teacher can proceed to highlight for the class effective strategies that this student employed. This might then further improve this student's experience with coding as well as their confidence. The teacher can subsequently reflect on how these interactions were helpful for this student as well as for their peers and imagine ways the experience could have been improved.

Building Healthy Relationships and Communicating Effectively in Mathematics

Scenario [supporting individual students within a group setting]: The teacher is presenting a mathematical modelling task that students are to complete in a small group.

Students' Initial Reaction: Some students express discomfort with working in a small group.

Teacher Prompt: The teacher checks in with these students to find out what makes them feel uncomfortable about working in a group. The students indicate that the lack of structure and accountability in group discussions and settings bothers them and makes them nervous about completing the task. Students may need support to describe what makes them uncomfortable.

Whole-Class Discussion: The teacher facilitates a discussion with the class to co-create working agreements that include guidelines and practices that are important to them in order to collaborate effectively as a group – providing brief examples of effective strategies that they already have seen from the students. Guidelines and practices could include listening attentively to each other's ideas; having one person at a time share their idea; ensuring that all ideas are respected and valued; deciding as a group which ideas for solving the task they will explore further. Students describe what these co-constructed agreements may look and sound like; for example, listening to ideas before commenting, or asking probing questions to clarify. The teacher records the agreements and posts them in the classroom as a reference for all group tasks.

Small-Group Discussion: The students begin working in small groups to complete the mathematical modelling task. The teacher visits each of the groups to monitor whether discussions are following the working agreements that were set by the class. The teacher checks in privately with the students who had expressed discomfort in group settings to see how they are feeling and identifies whether additional supports are needed in order for the students to be able to interact meaningfully with the group discussion. The teacher facilitates the discussion with each group by posing questions about the mathematical modelling task and supports students in using the guidelines that have been set.

Teacher Reflection [inform future action]: The teacher considers how these students responded to the discussion, and how future activities can be structured with a positive experience for these students in mind. The teacher reflects on how the guidelines established by the class can be revisited and further established in future conversations with students – recognizing that this is a fluid process.

Developing a Healthy Mathematical Identity Through Building Self-Awareness

Scenario [building empowerment through relevance using a strength-based approach]: Students are given the task of researching and telling a story about the development of a geometric concept that is relevant to them. The teacher provides guidance for a student who is unsure of how to approach this.

Small-Group Discussion: The teacher asks students to brainstorm in small groups some possible geometric concepts that they might be interested in researching and identify why each concept is of interest.

Teacher Prompt: The teacher asks students to select one of the concepts from their brainstorming session that might be relevant to them, in that they can make a personal connection to the concept or tell a story about it in a way that reflects their identities or interests.

Students' Action: Students gather information on their selected concepts. They may collect information from personal history, community organizations, or other resources.

Teacher Prompt: The teacher prompts students to decide how they would like to create and share their story about their chosen concept with others in the class. The teacher also asks the students to make connections to a career or to a discipline such as the arts (visual or media arts or dance).

Student and Teacher Discussion: For the student who was unsure of how to create and share their story, the teacher and student work together to identify an approach that uses the student's strengths and highlights the relevance of the concept they have chosen (e.g., telling their story through a musical composition or a visual display).

Teacher Reflection [connect the intent of the curriculum and various global perspectives]: The teacher reflects on how stories of geometric concepts were represented by various cultures demonstrates their significant contributions to mathematics. The teacher thinks about including the stories of various cultures in their teaching so that students come to understand and appreciate mathematics as a human story.

Developing Critical and Creative Mathematical Thinking

Scenario [supporting students who are working in pairs in developing an appreciation of other perspectives]: Students are given the task of creating a table of values, a graph, and an algebraic expression to represent a linear relation.

Students' Actions: One student chooses to start with creating a graph to represent their relation. Another student chooses to start with creating a table of values.

Teacher Facilitation: The teacher is monitoring the class as students are creating their representations and notices that two students are approaching the task by starting with different representations. The teacher checks with the students to see if they are okay with sharing with the rest of the class their work so far. The students agree, and the teacher pauses the class and invites the students to share. The teacher reinforces the idea that mathematicians use different representations as starting points in their problem solving and consolidates the critical understanding that different starting points and approaches can lead to the same result. These two students share their reasoning for why they chose to start with the representations that they did, and elaborate on how they made connections and decisions as part of their thinking process.

Students' Responses: The student who started with a graph shared that they like to see whether they are creating a line that has a positive slope or a negative slope. For them, a visual representation is a helpful starting point. The student who started with the table of values shared that they wanted to create their linear relation by seeing the pattern between each pair of points. For this student, being able to see the numbers side by side helps to show the relationship between the numbers.

Teacher Facilitation: The teacher encourages the rest of the students to think about why they chose the representation that they did, and then asks them to also think about the benefits of being aware of the reasons behind their decisions. The teacher also prompts a discussion about the connections among the different representations and how each of them provides information, such as comparing the amount the graph goes up or down at each iteration with the change in the value in the chart of the dependent variable. Next, this might also be compared to the slope, and the meanings all explicitly connected.

Teacher Reflection [consider the effectiveness of messaging]: The teacher reflects on the effectiveness of their comments about various representations – whether students connected their own experiences with the summary that the teacher shared when facilitating the discussion. The teacher may continue to reflect on the direction of future lessons, and on whether more activities are necessary or helpful for students to better appreciate the importance of connecting various representations.

Instructional tips

Approaches to Instruction of Social-Emotional Learning Skills
See the section Elements of the Grade 9 Mathematics Course – Social-Emotional Learning (SEL) Skills in Grade 9 Mathematics for essential information on approaches to instruction.

Opportunities for Planning Instruction
Teachers are encouraged to look for opportunities to highlight and embed explorations of and connections to social-emotional learning skills, where appropriate, within the learning throughout the course, in order to support students in developing and applying these skills.

When reviewing the **Instructional Tips** provided in each strand and planning for instruction throughout the course, teachers consider how to use strategies such as the sample strategies outlined in the section mentioned above to support the instruction of social-emotional learning skills in an inclusive way.

Below are some examples of opportunities that teachers might find for instruction of social-emotional learning skills as part of students' learning related to the other expectations throughout the course. Note that that there are many other possible opportunities for this to occur, taking into account students' strengths and needs and the learning context.

- **Recognizing and Identifying Emotions That Support Mathematical Learning**
  Through instruction, teachers can support students in:
    - recognizing that new or challenging learning may involve a sense of excitement or an initial sense of discomfort, and honouring those emotions as they arise; for example, when students are writing and altering code to represent mathematical situations (C2.1, C2.2).

Note

Engaging in self-reflection is key for teachers in recognizing that various experiences in mathematics will elicit a range of emotions from students. It is important for teachers to be responsive to emotions they observe in the students they are working with and work to avoid anticipating certain emotions based on personal experience. See [Social-Emotional Learning (SEL) Skills in Grade 9 Mathematics] for further guidance.

- **Recognizing Sources of Stress That Present Challenges to Mathematical Learning**
  Through instruction, teachers can support students in:
  - approaching peers, teachers, other staff, family, and/or their extended community for support in a range of situations; for example, when students are applying the process of mathematical modelling to solve real-life problems (D2.2, D2.3, D2.4, D2.5);
  - applying strategies such as engaging in guided imagery and visualization to help make mathematical connections; for example, when students are making connections between $y = ax$ and its various transformations (C4.3).
- **Identifying Resources and Supports That Aid Perseverance in Mathematical Learning**
  Through instruction, teachers can support students in:
  - recognizing mistakes as a necessary and helpful part of learning; for example, when students are solving problems involving conversions between different units or between measurement systems (E1.3);
  - encouraging students to persevere and seek support when they find concepts and exercises to be challenging;
  - noticing strengths and positive aspects of experiences, appreciating the value of practice and the necessity of repetition, and reflecting on the process of practice; for example, when students are solving problems involving operations with positive and negative fractions and mixed numbers (B3.4).
- **Building Healthy Relationships and Communicating Effectively in Mathematics**
  Through instruction, teachers can support students in:
  - listening attentively and respectfully in various situations; for example, when peers are sharing their stories about a mathematical concept of their interest, in order to understand and appreciate the perspectives of others' identities, knowledge, and experiences (B1.1, C1.1, E1.1);
  - considering other ideas and perspectives from peers, parents, and the wider community; for example, when students are sharing their rationales for budget modifications (F1.4).
- **Developing a Healthy Mathematical Identity Through Building Self-Awareness**
  Through instruction, teachers can support students in:

- identifying their personal strengths and exercising their own creativity as they engage in a variety of tasks; for example, when students are creating and analysing geometric designs that are relevant to them (E1.2);
- nurturing a sense of belonging and community; for example, when students are making connections between mathematics, various knowledge systems, and real-life applications of mathematics, by recognizing a range of experiences (A2).

- **Developing Critical and Creative Mathematical Thinking**
  Through instruction, teachers can support students in:
  - making connections between different forms of representation; for example, when students are comparing characteristics of graphs, tables of values, and equations of linear and non-linear relations (C4.1);
  - making decisions; for example, when students are considering how to represent and analyse data (D1.2).

Examples

The following examples illustrate various ways teachers may provide instruction to support students in developing social-emotional learning skills in connection to learning mathematics.

Recognizing and Identifying Emotions That Support Mathematical Learning

Scenario [checking observations with student]: A student has been given the task of multiplying powers with variable bases (e.g., $a^2 \cdot a^8$).

Teacher Observation: The teacher perceives that the student appears to be frustrated with the task.

Note: Each teacher's perspectives are subject to and informed by their own experiences. What the teacher observes and perceives may or may not align with what the student is actually feeling and experiencing. A culturally responsive and relevant approach starts with the teacher engaging in self-reflection, then considering elements of an inclusive learning environment and the educational context in which they are observing the student.

Teacher Action and Student Response: The teacher asks the student if they would like to talk through the task together. The student accepts. During the conversation, the teacher supports the student in identifying the emotions they are feeling and helps build the student's understanding that thoughts, feelings, and actions are all connected and affect one another. In this case, the student identifies that they are feeling confused and frustrated. The teacher works with the student to identify strategies that will help in this situation (e.g., identify what they do understand, develop related mathematical literacy, make connections by relating the task to what they know about powers with numeric bases, seek further information by reviewing class notes about exponent laws, explore different ways of looking at the problem, take a break and come back to the task later). In this case, the teacher asks the student to think about a similar situation and strategies they may have used when they multiplied powers with bases that were integers. For example, the student could be encouraged to write out the expanded form of the expression, without exponents, use that expression to recall what to do and why the method makes sense, then generalize this understanding and apply the method to the problem with the variable bases. In this way, the student makes the connection to their prior learning and applies it to complete the task. The student could then reflect on how the approach they took to work through this task might help them the next time they feel frustrated with a task.

Teacher Reflection [continue reflection on an ongoing basis]: The teacher reflects on how their actions may have affected the student's level of confidence in using strategies for continuing to problem solve when the student feels frustrated. The teacher thinks about potential alternatives for future interactions with this student. This interaction also supports teacher decisions on future tasks that may improve this particular student's perseverance skills and confidence.

Note: Ongoing teacher reflection is important throughout instruction, not just at the beginning and end. It includes developing an understanding of individual student identities, strengths, and needs, including language learning and educational experience. This is a critical first step in building trust and relationships with students.

Recognizing Sources of Stress That Present Challenges to Mathematical Learning

Scenario [using culturally responsive pedagogy to build a plan of action in response to individual student strengths and needs]: A student has been given a task that involves fractions.

Student's Initial Reaction: The student shares with the teacher that they get stressed every time they encounter a fraction.

Teacher and Student Conversation: The teacher thanks the student for sharing information about how they are feeling and asks if they would like to share more about why fractions create a stressful response for them. The student shares with the teacher that they do not see the relevance of fractions to their everyday life, and that they have always struggled with fractions for as long as they can remember.

The teacher acknowledges the student's feelings of stress, and helps the student respond to these feelings by identifying what they do know about fractions. Next steps could include identifying what additional support is needed to help the student feel less stressed and be more successful and confident when working with fractions. For example, the teacher could select mathematical tasks that are contextualized to provide a relatable entry point for this student, including the use of fraction manipulatives (e.g., fraction strips, relational rods) to complete the tasks.

Teacher Reflection [consider strengths and needs of individual students]: The teacher thinks about how their approach to working with individual students to identify personal strategies to respond to stress seemed to have been received by this particular student. This provides useful information about strategies that might best support this student in the future.

Identifying Resources and Supports That Aid Perseverance in Mathematical Learning

Scenario [providing one-on-one support]: A student has been given the task of writing code to explore what happens to the volume of a rectangular prism when one, two, and three dimensions of the prism are altered.

Student's Initial Reaction: The student shares with the teacher that they do not know how to begin the task and they feel overwhelmed.

Teacher's Response to Student: The teacher has a conversation with the student to learn about their prior experience with coding. The teacher then works directly with the student, supporting them as they develop a physical model and a flow chart to plan out the code for exploring what happens to the volume of a rectangular prism when one dimension changes. Next, the teacher has the student write the code and execute it to see if they get the output they were expecting.

Student's Response to Teacher: The student shares that the code they wrote produced what they expected.

Teacher's Response to Student: The teacher then works with the student to identify how the model and the flow chart will need to be altered to reflect a change in two dimensions instead of one. The teacher points out that knowing the steps to follow and knowing that it is okay if the results aren't as expected both aid in persevering with the task.

Student's Response to Teacher: The student shares with the teacher that they now know what to do and can proceed with writing the remaining code to complete the task.

After getting to this stage, the student reflects that one of the benefits of coding is that one can get feedback right away, and that it is okay if the results are not as expected. Identifying the reason for the difference is part of the process and the challenge.

Teacher Reflection [consider strategies for uplifting students and impact of this]: The teacher thinks about aspects of what this student did that could be helpful to other students. After inquiring whether this student is willing to share their successes, the teacher can proceed to highlight for the class effective strategies that this student employed. This might then further improve this student's experience with coding as well as their confidence. The teacher can subsequently reflect on how these interactions were helpful for this student as well as for their peers and imagine ways the experience could have been improved.

Building Healthy Relationships and Communicating Effectively in Mathematics

Scenario [supporting individual students within a group setting]: The teacher is presenting a mathematical modelling task that students are to complete in a small group.

Students' Initial Reaction: Some students express discomfort with working in a small group.

Teacher Prompt: The teacher checks in with these students to find out what makes them feel uncomfortable about working in a group. The students indicate that the lack of structure and accountability in group discussions and settings bothers them and makes them nervous about completing the task. Students may need support to describe what makes them uncomfortable.

Whole-Class Discussion: The teacher facilitates a discussion with the class to co-create working agreements that include guidelines and practices that are important to them in order to collaborate effectively as a group – providing brief examples of effective strategies that they already have seen from the students. Guidelines and practices could include listening attentively to each other's ideas; having one person at a time share their idea; ensuring that all ideas are respected and valued; deciding as a group which ideas for solving the task they will explore further. Students describe what these co-constructed agreements may look and sound like; for example, listening to ideas before commenting, or asking probing questions to clarify. The teacher records the agreements and posts them in the classroom as a reference for all group tasks.

Small-Group Discussion: The students begin working in small groups to complete the mathematical modelling task. The teacher visits each of the groups to monitor whether discussions are following the working agreements that were set by the class. The teacher checks in privately with the students who had expressed discomfort in group settings to see how they are feeling and identifies whether additional supports are needed in order for the students to be able to interact meaningfully with the group discussion. The teacher facilitates the discussion with each group by posing questions about the mathematical modelling task and supports students in using the guidelines that have been set.

Teacher Reflection [inform future action]: The teacher considers how these students responded to the discussion, and how future activities can be structured with a positive experience for these students in mind. The teacher reflects on how the guidelines established by the class can be revisited and further established in future conversations with students – recognizing that this is a fluid process.

Developing a Healthy Mathematical Identity Through Building Self-Awareness

Scenario [building empowerment through relevance using a strength-based approach]: Students are given the task of researching and telling a story about the development of a geometric concept that is relevant to them. The teacher provides guidance for a student who is unsure of how to approach this.

Small-Group Discussion: The teacher asks students to brainstorm in small groups some possible geometric concepts that they might be interested in researching and identify why each concept is of interest.

Teacher Prompt: The teacher asks students to select one of the concepts from their brainstorming session that might be relevant to them, in that they can make a personal connection to the concept or tell a story about it in a way that reflects their identities or interests.

Students' Action: Students gather information on their selected concepts. They may collect information from personal history, community organizations, or other resources.

Teacher Prompt: The teacher prompts students to decide how they would like to create and share their story about their chosen concept with others in the class. The teacher also asks the students to make connections to a career or to a discipline such as the arts (visual or media arts or dance).

Student and Teacher Discussion: For the student who was unsure of how to create and share their story, the teacher and student work together to identify an approach that uses the student's strengths and highlights the relevance of the concept they have chosen (e.g., telling their story through a musical composition or a visual display).

Teacher Reflection [connect the intent of the curriculum and various global perspectives]: The teacher reflects on how stories of geometric concepts were represented by various cultures demonstrates their significant contributions to mathematics. The teacher thinks about including the stories of various cultures in their teaching so that students come to understand and appreciate mathematics as a human story.

Developing Critical and Creative Mathematical Thinking

Scenario [supporting students who are working in pairs in developing an appreciation of other perspectives]: Students are given the task of creating a table of values, a graph, and an algebraic expression to represent a linear relation.

Students' Actions: One student chooses to start with creating a graph to represent their relation. Another student chooses to start with creating a table of values.

Teacher Facilitation: The teacher is monitoring the class as students are creating their representations and notices that two students are approaching the task by starting with different representations. The teacher checks with the students to see if they are okay with sharing with the rest of the class their work so far. The students agree, and the teacher pauses the class and invites the students to share. The teacher reinforces the idea that mathematicians use different representations as starting points in their problem solving and consolidates the critical understanding that different starting points and approaches can lead to the same result. These two students share their reasoning for why they chose to start with the representations that they did, and elaborate on how they made connections and decisions as part of their thinking process.

Students' Responses: The student who started with a graph shared that they like to see whether they are creating a line that has a positive slope or a negative slope. For them, a visual representation is a helpful starting point. The student who started with the table of values shared that they wanted to create their linear relation by seeing the pattern between each pair of points. For this student, being able to see the numbers side by side helps to show the relationship between the numbers.

Teacher Facilitation: The teacher encourages the rest of the students to think about why they chose the representation that they did, and then asks them to also think about the benefits of being aware of the reasons behind their decisions. The teacher also prompts a discussion about the connections among the different representations and how each of them provides information, such as comparing the amount the graph goes up or down at each iteration with the change in the value in the chart of the dependent variable. Next, this might also be compared to the slope, and the meanings all explicitly connected.

Teacher Reflection [consider the effectiveness of messaging]: The teacher reflects on the effectiveness of their comments about various representations – whether students connected their own experiences with the summary that the teacher shared when facilitating the discussion. The teacher may continue to reflect on the direction of future lessons, and on whether more activities are necessary or helpful for students to better appreciate the importance of connecting various representations.

Instructional tips

Approaches to Instruction of Social-Emotional Learning Skills
See the section Elements of the Grade 9 Mathematics Course – Social-Emotional Learning (SEL) Skills in Grade 9 Mathematics for essential information on approaches to instruction.

Opportunities for Planning Instruction
Teachers are encouraged to look for opportunities to highlight and embed explorations of and connections to social-emotional learning skills, where appropriate, within the learning throughout the course, in order to support students in developing and applying these skills.

When reviewing the **Instructional Tips** provided in each strand and planning for instruction throughout the course, teachers consider how to use strategies such as the sample strategies outlined in the section mentioned above to support the instruction of social-emotional learning skills in an inclusive way.

Below are some examples of opportunities that teachers might find for instruction of social-emotional learning skills as part of students' learning related to the other expectations throughout the course. Note that that there are many other possible opportunities for this to occur, taking into account students' strengths and needs and the learning context.

- **Recognizing and Identifying Emotions That Support Mathematical Learning**
  Through instruction, teachers can support students in:

- recognizing that new or challenging learning may involve a sense of excitement or an initial sense of discomfort, and honouring those emotions as they arise; for example, when students are writing and altering code to represent mathematical situations (C2.1, C2.2).

Note

Engaging in self-reflection is key for teachers in recognizing that various experiences in mathematics will elicit a range of emotions from students. It is important for teachers to be responsive to emotions they observe in the students they are working with and work to avoid anticipating certain emotions based on personal experience. See Social-Emotional Learning (SEL) Skills in Grade 9 Mathematics for further guidance.

- **Recognizing Sources of Stress That Present Challenges to Mathematical Learning**
  Through instruction, teachers can support students in:
  - approaching peers, teachers, other staff, family, and/or their extended community for support in a range of situations; for example, when students are applying the process of mathematical modelling to solve real-life problems (D2.2, D2.3, D2.4, D2.5);
  - applying strategies such as engaging in guided imagery and visualization to help make mathematical connections; for example, when students are making connections between $y = ax$ and its various transformations (C4.3).
- **Identifying Resources and Supports That Aid Perseverance in Mathematical Learning**
  Through instruction, teachers can support students in:
  - recognizing mistakes as a necessary and helpful part of learning; for example, when students are solving problems involving conversions between different units or between measurement systems (E1.3);
  - encouraging students to persevere and seek support when they find concepts and exercises to be challenging;
  - noticing strengths and positive aspects of experiences, appreciating the value of practice and the necessity of repetition, and reflecting on the process of practice; for example, when students are solving problems involving operations with positive and negative fractions and mixed numbers (B3.4).
- **Building Healthy Relationships and Communicating Effectively in Mathematics**
  Through instruction, teachers can support students in:
  - listening attentively and respectfully in various situations; for example, when peers are sharing their stories about a mathematical concept of their interest, in order to understand and appreciate the perspectives of others' identities, knowledge, and experiences (B1.1, C1.1, E1.1);
  - considering other ideas and perspectives from peers, parents, and the wider community; for example, when students are sharing their rationales for budget modifications (F1.4).
- **Developing a Healthy Mathematical Identity Through Building Self-Awareness**
  Through instruction, teachers can support students in:

    - identifying their personal strengths and exercising their own creativity as they engage in a variety of tasks; for example, when students are creating and analysing geometric designs that are relevant to them (E1.2);
    - nurturing a sense of belonging and community; for example, when students are making connections between mathematics, various knowledge systems, and real-life applications of mathematics, by recognizing a range of experiences (A2).
- **Developing Critical and Creative Mathematical Thinking**
  Through instruction, teachers can support students in:
    - making connections between different forms of representation; for example, when students are comparing characteristics of graphs, tables of values, and equations of linear and non-linear relations (C4.1);
    - making decisions; for example, when students are considering how to represent and analyse data (D1.2).

A. Mathematical Thinking and Making Connections

This strand has no specific expectations. Students' learning related to this strand takes place in the context of learning related to strands B through F, and it should be assessed and evaluated within these contexts.

Overall expectations

Throughout this course, in connection with the learning in the other strands, students will:

A1. Mathematical Processes

apply the mathematical processes to develop a conceptual understanding of, and procedural fluency with, the mathematics they are learning

Teacher supports

Instructional tips

Teachers can:

- highlight the interconnectedness of the seven mathematical processes (problem solving, reasoning and proving, reflecting, connecting, communicating, representing, and selecting tools and strategies) and model ways for students to combine them while doing mathematics;

- support students in activating their prior knowledge when encountering new concepts *(connecting)*;
- provide students with opportunities to integrate their learning within and across the strands, explicitly demonstrating and reinforcing connections between various mathematical concepts *(connecting)*;
- support students' appreciation of the innate beauty of mathematics *(problem solving, reasoning and proving, reflecting, connecting, communicating, representing, selecting tools and strategies)*;
- pose problems that have multiple entry points and can be solved in various ways *(problem solving)*;
- provide students with opportunities to pose and solve authentic problems that are of interest to them *(problem solving)*;
- make available a range of materials and technologies for students to choose from and teach them how to select and use tools to represent mathematical situations, solve problems, and communicate their thinking *(selecting tools and strategies)*;
- support students in understanding that the same mathematical situation can be represented in various ways, and in making connections among different representations *(representing, connecting)*;
- empower students to think about and strategize how they solve problems, including steps such as representing the situations, selecting tools and strategies, reflecting on the reasonableness of their solutions, and justifying their thinking *(problem solving, representing, selecting tools and strategies, reflecting, reasoning and proving)*;
- encourage students to reflect on their mistakes and on feedback they are given and to revise their mathematical solutions as necessary, demonstrating how doing so can move learning forward *(reflecting)*;
- provide opportunities for students to respectfully listen, reflect, and discuss strategies and reasoning in pairs or small groups *(communicating, reflecting, reasoning and proving)*;
- facilitate the purposeful sharing of different problem-solving strategies for the same problem, including validating, recognizing, and encouraging fruitful aspects in each student's strategy *(communicating, reflecting)*;
- support all students in expanding their communicative repertoire to include a broader range of terminology and conventions *(communicating)*;
- create opportunities for meaningful peer feedback and emphasize the benefits of discourse in the learning of mathematics *(communicating)*.

Teacher prompts

Prompts that highlight specific mathematical processes

Problem Solving

- Explain in your own words the problem you need to solve.
- What information, knowledge, and strategies may be helpful to solve this problem?
- What assumptions are inherent in the problem? What assumptions are you making?
- What information do you know already and what additional information is required to solve the problem?

Reasoning and Proving

- Is this statement true for all cases?
- How can you verify this answer?
- What would happen if …? (e.g., if the rate of change increased? if this number were negative?)
- How can you extend these ideas to more general cases?

Reflecting

- Does this answer make sense to you? Why or why not?
- How did the learning tool you chose contribute to your understanding of, or solution for, the problem?
- How did the tool assist you in communicating your answer or your thinking?
- What strategies did you use that did or did not work?
- What did you learn in the process of working through this problem?

Connecting

- What connection(s) do you see between a problem you solved previously [describe the problem student solved previously] and today's problem?
- Describe the connections you see between … and …. (e.g., one representation with another, a student's interpretation with another, a student's strategy with another)
- How does your representation (e.g., diagram, sketch, concrete/digital representation) connect to …? (e.g., the algebraic solution? another student's work?)
- How is the strategy that was just shared in the group discussion similar to or different from your strategy?

Communicating

- Present your solution to a problem so that someone else will understand your thinking and your process.
- Share your thinking with this group and consider their feedback as you revise your work.
- How can you express this in another way?

Representing

- In what other way(s) can you represent this situation?
- How could you represent this situation using a graph? using a diagram? using concrete/digital tools? using a table of values?
- In what way(s) would a scale model help you solve this problem?

Selecting Tools and Strategies

- Explain why you chose to use this tool/strategy to solve the problem.
- What other tools/strategies did you consider using? Explain why you chose not to use them.
- What were the advantages and disadvantages of the strategies you tried?
- What estimation strategy did you use? Was your result sufficiently accurate for the situation?

Sample tasks

Sample tasks from Strands B to F that highlight various mathematical processes:

Number

B2.1 Sample task that highlights reasoning and proving, and selecting tools and strategies:

- Have students determine, using a strategy of their choice, which number is smaller, -4×10^3 or 4×10^{-3}.

Algebra

C4.2 Sample task that highlights representing and making connections:

- Ask students to generate a set of coordinates that satisfy the equation $x + y = 10$. Have them plot these coordinates on a grid, and then discuss whether they have found all the possible values that satisfy the equation or if there are others between these points. This discussion should lead to the idea of connecting the points with a line to represent all possible values. Then have them choose points above and below the line they have drawn, and ask them how these points are connected to the inequalities $x + y > 10$ and $x + y < 10$.

Data

D2.5 Teacher prompts that highlight reflecting (after completing a mathematical modelling task):

- Does your model help you to answer your question? Did you need to revise the model? Why?
- Does your model allow you to make predictions?
- What predictions can be made based on the model?
- What are the limits of the model?

Geometry and Measurement

E1.3 Sample task that highlights problem solving:

- Have students solve problems that involve using unconventional units to measure. For example: How many of the same type of coin are needed to go around the circumference of Earth?

Financial Literacy

F1.4 Sample task that highlights communication:

- Show students the budget for a division of the local municipal government (e.g., Parks and Recreation). Pose a scenario that is relevant to the current local situation (e.g., community members would like an outdoor skating rink) and have students discuss how the budget could be modified based on this scenario.

Instructional tips

Teachers can:

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- support students in activating their prior knowledge when encountering new concepts *(connecting)*;
- provide students with opportunities to integrate their learning within and across the strands, explicitly demonstrating and reinforcing connections between various mathematical concepts *(connecting)*;
- support students' appreciation of the innate beauty of mathematics *(problem solving, reasoning and proving, reflecting, connecting, communicating, representing, selecting tools and strategies)*;
- pose problems that have multiple entry points and can be solved in various ways *(problem solving)*;
- provide students with opportunities to pose and solve authentic problems that are of interest to them *(problem solving)*;
- make available a range of materials and technologies for students to choose from and teach them how to select and use tools to represent mathematical situations, solve problems, and communicate their thinking *(selecting tools and strategies)*;
- support students in understanding that the same mathematical situation can be represented in various ways, and in making connections among different representations *(representing, connecting)*;
- empower students to think about and strategize how they solve problems, including steps such as representing the situations, selecting tools and strategies, reflecting on the reasonableness of their solutions, and justifying their thinking *(problem solving, representing, selecting tools and strategies, reflecting, reasoning and proving)*;
- encourage students to reflect on their mistakes and on feedback they are given and to revise their mathematical solutions as necessary, demonstrating how doing so can move learning forward *(reflecting)*;
- provide opportunities for students to respectfully listen, reflect, and discuss strategies and reasoning in pairs or small groups *(communicating, reflecting, reasoning and proving)*;
- facilitate the purposeful sharing of different problem-solving strategies for the same problem, including validating, recognizing, and encouraging fruitful aspects in each student's strategy *(communicating, reflecting)*;
- support all students in expanding their communicative repertoire to include a broader range of terminology and conventions *(communicating)*;
- create opportunities for meaningful peer feedback and emphasize the benefits of discourse in the learning of mathematics *(communicating)*.

Teacher prompts

Prompts that highlight specific mathematical processes

Problem Solving

- Explain in your own words the problem you need to solve.
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- Is this statement true for all cases?
- How can you verify this answer?
- What would happen if …? (e.g., if the rate of change increased? if this number were negative?)
- How can you extend these ideas to more general cases?

Reflecting

- Does this answer make sense to you? Why or why not?
- How did the learning tool you chose contribute to your understanding of, or solution for, the problem?
- How did the tool assist you in communicating your answer or your thinking?
- What strategies did you use that did or did not work?
- What did you learn in the process of working through this problem?

Connecting

- What connection(s) do you see between a problem you solved previously [describe the problem student solved previously] and today's problem?
- Describe the connections you see between … and …. (e.g., one representation with another, a student's interpretation with another, a student's strategy with another)
- How does your representation (e.g., diagram, sketch, concrete/digital representation) connect to …? (e.g., the algebraic solution? another student's work?)
- How is the strategy that was just shared in the group discussion similar to or different from your strategy?

Communicating

- Present your solution to a problem so that someone else will understand your thinking and your process.

- Share your thinking with this group and consider their feedback as you revise your work.
- How can you express this in another way?

Representing

- In what other way(s) can you represent this situation?
- How could you represent this situation using a graph? using a diagram? using concrete/digital tools? using a table of values?
- In what way(s) would a scale model help you solve this problem?

Selecting Tools and Strategies

- Explain why you chose to use this tool/strategy to solve the problem.
- What other tools/strategies did you consider using? Explain why you chose not to use them.
- What were the advantages and disadvantages of the strategies you tried?
- What estimation strategy did you use? Was your result sufficiently accurate for the situation?

Sample tasks

Sample tasks from Strands B to F that highlight various mathematical processes:

Number

B2.1 Sample task that highlights reasoning and proving, and selecting tools and strategies:

- Have students determine, using a strategy of their choice, which number is smaller, -4×10^3 or 4×10^{-3}.

Algebra

C4.2 Sample task that highlights representing and making connections:

- Ask students to generate a set of coordinates that satisfy the equation $x + y = 10$. Have them plot these coordinates on a grid, and then discuss whether they have found all the possible values that satisfy the equation or if there are others between these points. This discussion should lead to the idea of connecting the points with a line to represent all possible values. Then have them choose points above and below the line they have drawn, and ask them how these points are connected to the inequalities $x + y > 10$ and $x + y < 10$.

Data

D2.5 Teacher prompts that highlight reflecting (after completing a mathematical modelling task):

- Does your model help you to answer your question? Did you need to revise the model? Why?
- Does your model allow you to make predictions?
- What predictions can be made based on the model?
- What are the limits of the model?

Geometry and Measurement

E1.3 Sample task that highlights problem solving:

- Have students solve problems that involve using unconventional units to measure. For example: How many of the same type of coin are needed to go around the circumference of Earth?

Financial Literacy

F1.4 Sample task that highlights communication:

- Show students the budget for a division of the local municipal government (e.g., Parks and Recreation). Pose a scenario that is relevant to the current local situation (e.g., community members would like an outdoor skating rink) and have students discuss how the budget could be modified based on this scenario.

A2. Making Connections

make connections between mathematics and various knowledge systems, their lived experiences, and various real-life applications of mathematics, including careers

Teacher supports

Instructional tips

Teachers can:

- demonstrate that they honour and value connections identified by students, and share their excitement over the connections they discover;
- build on students' sense of their identities, experiences in school and other experiences, ideas, questions, and interests to support the development of an engaging and inclusive mathematics learning community;
- create opportunities for every student to feel that they are reflected in mathematical learning;
- facilitate discussions about the applications of mathematics in the world outside the classroom, including the natural world;
- provide opportunities for students to recognize that mathematical knowledge has been developed by every culture around the globe;
- respectfully incorporate, in partnership through community connections (e.g., a community member, Elder, knowledge holder, or other individual with expertise), specific examples that highlight First Nations, Métis, and Inuit cultures and ways of being and knowing, in order to infuse Indigenous knowledges and perspectives meaningfully and authentically into the mathematics program;
- facilitate student-generated class discussions about careers and fields of study that students are interested in exploring.

Note

Teachers are encouraged to collaborate with community partners to plan culturally responsive and relevant teaching that honours and respects students' identities, and that includes real-life applications of mathematics that are relevant to students' lives and their communities.

Teacher prompts

- How does this mathematical concept connect to a story that has been shared by others in the class?
- How does this mathematical concept connect to something you have experienced in your life?
- How do you see this mathematical concept applied in real life?
- What are some careers that may use this mathematical concept, and in what ways?

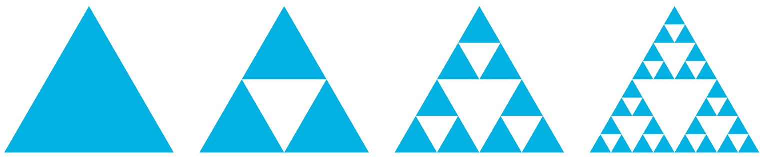
Sample tasks

Connections students may make when completing sample tasks for strands B to F:

Number

B1.3 Sample Task

Have students make their own fractal triangles (Sierpinski triangles) by repeating a simple pattern of triangles to create a complex image, then use these images to create as many kinds of different number sets as they can (e.g., the number of blue triangles in each image, the fraction of blue triangles to white triangles in each image) or another similar fractal.



A student engaged in this task might make connections between the concept of infinity in their fractal designs and the fractal designs they have seen in nature, such as in pine cones, ice crystals, and trees.

Algebra

C3.2 Sample Task

Provide students with one representation of a linear relation in context, and ask them to create a different representation of the relation. Some examples of contexts that might be relevant to students' lives are:

- cost of participating in various classes (e.g., dance, yoga, martial arts, fitness, music)
- distance travelled over time
- number of hours worked and total pay
- mass of bulk goods purchased and cost
- area of land and crop yield

A student engaged in this task might make connections between a real-life context that is familiar to them and the way each type of representation reveals information about that context.

Data

D2.1 Sample Task

Have students research careers that involve mathematical modelling.

A student engaged in this task might make connections between mathematical modelling and careers such as game designer, meteorologist, actuary, marketing analyst, and biostatistician.

Geometry and Measurement

E1.3 Sample Task

Ask students to share a recipe that is relevant to them, their family, and/or their community. Have them exchange recipes and pose questions for other students to answer, such as:

- What ingredient do you need the most of, and how can you tell?
- If you are missing one of the measuring tools, how can you use another measuring tool to measure the appropriate amount?
- If the recipe uses mass, how can you convert it to use capacity measuring tools?

A student engaged in this task might make connections between recipes that use imperial measures, such as cups, those that use metric measures, such as grams, and other types of measures, such as handfuls.

Financial Literacy

F1.2 Sample Task

Have students brainstorm examples of assets that appreciate or depreciate, including those that might experience a short-term appreciation due to a current trend (e.g., trading cards, trends started on social media). Show students graphs or provide them with data to graph depicting the appreciation and depreciation of assets identified during the brainstorming session and have them identify what they notice and what questions they might still have about each of the graphs.

A student engaged in this task might make connections between assets that appreciate or depreciate in value and items they are collecting, such as video games, comic books, or sports memorabilia.

Instructional tips

Teachers can:

- demonstrate that they honour and value connections identified by students, and share their excitement over the connections they discover;
- build on students' sense of their identities, experiences in school and other experiences, ideas, questions, and interests to support the development of an engaging and inclusive mathematics learning community;
- create opportunities for every student to feel that they are reflected in mathematical learning;
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Teacher prompts

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- What are some careers that may use this mathematical concept, and in what ways?

Sample tasks

Connections students may make when completing sample tasks for strands B to F:

Number

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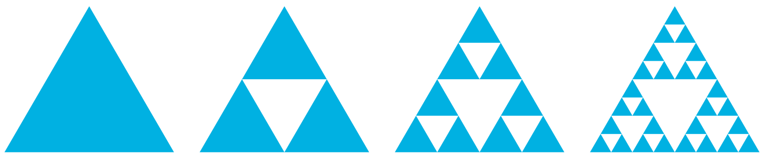
Have students make their own fractal triangles (Sierpinski triangles) by repeating a simple pattern of triangles to create a complex image, then use these images to create as many kinds of different number sets as they can (e.g., the number of blue triangles in each image, the fraction of blue triangles to white triangles in each image) or another similar fractal.



A student engaged in this task might make connections between the concept of infinity in their fractal designs and the fractal designs they have seen in nature, such as in pine cones, ice crystals, and trees.

Algebra

C3.2 Sample Task

Provide students with one representation of a linear relation in context, and ask them to create a different representation of the relation. Some examples of contexts that might be relevant to students' lives are:

- cost of participating in various classes (e.g., dance, yoga, martial arts, fitness, music)
- distance travelled over time
- number of hours worked and total pay
- mass of bulk goods purchased and cost
- area of land and crop yield

A student engaged in this task might make connections between a real-life context that is familiar to them and the way each type of representation reveals information about that context.

Data

D2.1 Sample Task

Have students research careers that involve mathematical modelling.

A student engaged in this task might make connections between mathematical modelling and careers such as game designer, meteorologist, actuary, marketing analyst, and biostatistician.

Geometry and Measurement

E1.3 Sample Task

Ask students to share a recipe that is relevant to them, their family, and/or their community. Have them exchange recipes and pose questions for other students to answer, such as:

- What ingredient do you need the most of, and how can you tell?
- If you are missing one of the measuring tools, how can you use another measuring tool to measure the appropriate amount?
- If the recipe uses mass, how can you convert it to use capacity measuring tools?

A student engaged in this task might make connections between recipes that use imperial measures, such as cups, those that use metric measures, such as grams, and other types of measures, such as handfuls.

Financial Literacy

F1.2 Sample Task

Have students brainstorm examples of assets that appreciate or depreciate, including those that might experience a short-term appreciation due to a current trend (e.g., trading cards, trends started on social media). Show students graphs or provide them with data to graph depicting the appreciation and depreciation of assets identified during the brainstorming session and have them identify what they notice and what questions they might still have about each of the graphs.

A student engaged in this task might make connections between assets that appreciate or depreciate in value and items they are collecting, such as video games, comic books, or sports memorabilia.

B. Number

Overall expectations

By the end of this course, students will:

B1. Development of Numbers and Number Sets

demonstrate an understanding of the development and use of numbers, and make connections between sets of numbers

Specific expectations

By the end of this course, students will:

Development and Use of Numbers

B1.1 research a number concept to tell a story about its development and use in a specific culture, and describe its relevance in a current context

Number Sets

B1.2 describe how various subsets of a number system are defined, and describe similarities and differences between these subsets

B1.3 use patterns and number relationships to explain density, infinity, and limit as they relate to number sets

B2. Powers

represent numbers in various ways, evaluate powers, and simplify expressions by using the relationships between powers and their exponents

Specific expectations

By the end of this course, students will:

Powers

B2.1 analyse, through the use of patterning, the relationship between the sign and size of an exponent and the value of a power, and use this relationship to express numbers in scientific notation and evaluate powers

B2.2 analyse, through the use of patterning, the relationships between the exponents of powers and the operations with powers, and use these relationships to simplify numeric and algebraic expressions

B3. Number Sense and Operations

apply an understanding of rational numbers, ratios, rates, percentages, and proportions, in various mathematical contexts, and to solve problems

Specific expectations

By the end of this course, students will:

Rational Numbers

B3.1 apply an understanding of integers to describe location, direction, amount, and changes in any of these, in various contexts

B3.2 apply an understanding of unit fractions and their relationship to other fractional amounts, in various contexts, including the use of measuring tools

B3.3 apply an understanding of integers to explain the effects that positive and negative signs have on the values of ratios, rates, fractions, and decimals, in various contexts

Applications

B3.4 solve problems involving operations with positive and negative fractions and mixed numbers, including problems involving formulas, measurements, and linear relations, using technology when appropriate

B3.5 pose and solve problems involving rates, percentages, and proportions in various contexts, including contexts connected to real-life applications of data, measurement, geometry, linear relations, and financial literacy

C. Algebra

Overall expectations

By the end of this course, students will:

C1. Algebraic Expressions and Equations

demonstrate an understanding of the development and use of algebraic concepts and of their connection to numbers, using various tools and representations

Specific expectations

By the end of this course, students will:

Development and Use of Algebra

C1.1 research an algebraic concept to tell a story about its development and use in a specific culture, and describe its relevance in a current context

Algebraic Expressions and Equations

C1.2 create algebraic expressions to generalize relationships expressed in words, numbers, and visual representations, in various contexts

C1.3 compare algebraic expressions using concrete, numerical, graphical, and algebraic methods to identify those that are equivalent, and justify their choices

C1.4 simplify algebraic expressions by applying properties of operations of numbers, using various representations and tools, in different contexts

C1.5 create and solve equations for various contexts, and verify their solutions

C2. Coding

apply coding skills to represent mathematical concepts and relationships dynamically, and to solve problems, in algebra and across the other strands

Specific expectations

By the end of this course, students will:

Coding

C2.1 use coding to demonstrate an understanding of algebraic concepts including variables, parameters, equations, and inequalities

C2.2 create code by decomposing situations into computational steps in order to represent mathematical concepts and relationships, and to solve problems

C2.3 read code to predict its outcome, and alter code to adjust constraints, parameters, and outcomes to represent a similar or new mathematical situation

C3. Application of Relations

represent and compare linear and non-linear relations that model real-life situations, and use these representations to make predictions

Specific expectations

By the end of this course, students will:

Application of Linear and Non-Linear Relations

C3.1 compare the shapes of graphs of linear and non-linear relations to describe their rates of change, to make connections to growing and shrinking patterns, and to make predictions

C3.2 represent linear relations using concrete materials, tables of values, graphs, and equations, and make connections between the various representations to demonstrate an understanding of rates of change and initial values

C3.3 compare two linear relations of the form $y = ax + b$ graphically and algebraically, and interpret the meaning of their point of intersection in terms of a given context

C4. Characteristics of Relations

demonstrate an understanding of the characteristics of various representations of linear and non-linear relations, using tools, including coding when appropriate

Specific expectations

By the end of this course, students will:

Characteristics of Linear and Non-Linear Relations

C4.1 compare characteristics of graphs, tables of values, and equations of linear and non-linear relations

C4.2 graph relations represented as algebraic equations of the forms x = k, y = k, $x + y$ = k, $x - y$ = k, ax + by = k, and xy = k, and their associated inequalities, where a, b, and k are constants, to identify various characteristics and the points and/or regions defined by these equations and inequalities

C4.3 translate, reflect, and rotate lines defined by y = ax, where a is a constant, and describe how each transformation affects the graphs and equations of the defined lines

C4.4 determine the equations of lines from graphs, tables of values, and concrete representations of linear relations by making connections between rates of change and slopes, and between initial values and y-intercepts, and use these equations to solve problems

D. Data

Overall expectations

By the end of this course, students will:

D1. Collection, Representation, and Analysis of Data

describe the collection and use of data, and represent and analyse data involving one and two variables

Specific expectations

By the end of this course, students will:

Application of Data

D1.1 identify a current context involving a large amount of data, and describe potential implications and consequences of its collection, storage, representation, and use

Representation and Analysis of Data

D1.2 represent and statistically analyse data from a real-life situation involving a single variable in various ways, including the use of quartile values and box plots

D1.3 create a scatter plot to represent the relationship between two variables, determine the correlation between these variables by testing different regression models using technology, and use a model to make predictions when appropriate

D2. Mathematical Modelling

apply the process of mathematical modelling, using data and mathematical concepts from other strands, to represent, analyse, make predictions, and provide insight into real-life situations

The Mathematical Modelling Process

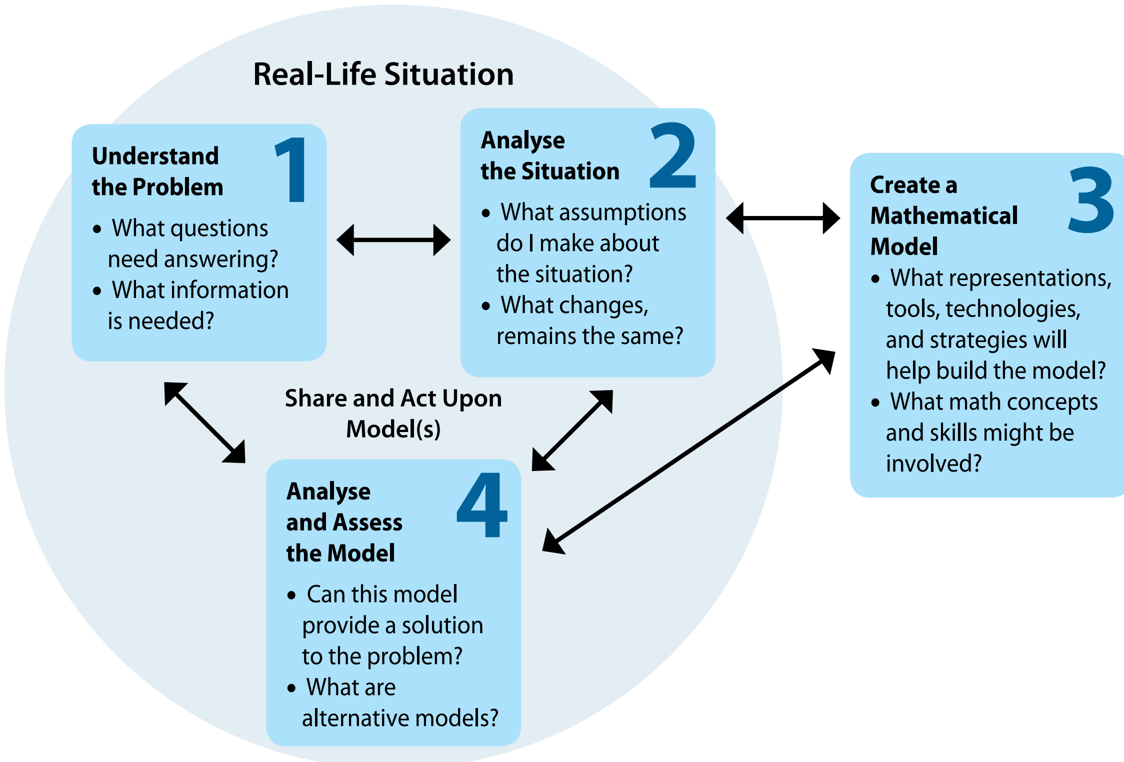
Mathematical modelling provides authentic connections to real-life situations. The process starts with ill-defined, often messy real-life problems that may have several different solutions that are all correct. Mathematical modelling requires the modeller to be critical and creative and make choices, assumptions, and decisions. Through this process, they create a mathematical model that describes a situation using mathematical concepts and language, and that can be used to solve a problem or make decisions and can be used to deepen understanding of mathematical concepts.

The process of mathematical modelling[13] has four key components that are interconnected and applied in an iterative way, where students may move between and across, as well as return to, each of the four components as they change conditions to observe new outcomes until the model is ready to be shared and acted upon. While moving through these components, social-emotional learning skills and mathematical processes are applied as needed.

1. Understand the problem
   - What questions need answering?
   - What information is needed?
2. Analyse the situation
   - What assumptions do I make about the situation?
   - What changes, what remains the same?
3. Create a mathematical model
   - What representations, tools, technologies, and strategies will help build the model?
   - What mathematical knowledge, concepts, and skills might be involved?
4. Analyse and assess the model
   - Can this model provide a solution?
   - What are alternative models?

[13] Hirsch, C. R., & McDuffie, A. R. (Eds.). (2016). *Annual perspectives in mathematics education 2016: Mathematical modeling and modeling mathematics.* Reston, VA: National Council of Teachers of Mathematics.

The Process of Mathematical Modelling



Specific expectations

By the end of this course, students will:

Application of Mathematical Modelling

D2.1 describe the value of mathematical modelling and how it is used in real life to inform decisions

Process of Mathematical Modelling

D2.2 identify a question of interest requiring the collection and analysis of data, and identify the information needed to answer the question

D2.3 create a plan to collect the necessary data on the question of interest from an appropriate source, identify assumptions, identify what may vary and what may remain the same in the situation, and then carry out the plan

D2.4 determine ways to display and analyse the data in order to create a mathematical model to answer the original question of interest, taking into account the nature of the data, the context, and the assumptions made

D2.5 report how the model can be used to answer the question of interest, how well the model fits the context, potential limitations of the model, and what predictions can be made based on the model

E. Geometry and Measurement

Overall expectations

By the end of this course, students will:

E1. Geometric and Measurement Relationships

demonstrate an understanding of the development and use of geometric and measurement relationships, and apply these relationships to solve problems, including problems involving real-life situations

Specific expectations

By the end of this course, students will:

Geometric and Measurement Relationships

E1.1 research a geometric concept or a measurement system to tell a story about its development and use in a specific culture or community, and describe its relevance in connection to careers and to other disciplines

E1.2 create and analyse designs involving geometric relationships and circle and triangle properties, using various tools

E1.3 solve problems involving different units within a measurement system and between measurement systems, including those from various cultures or communities, using various representations and technology, when appropriate

E1.4 show how changing one or more dimensions of a two-dimensional shape and a three-dimensional object affects perimeter/circumference, area, surface area, and volume, using technology when appropriate

E1.5 solve problems involving the side-length relationship for right triangles in real-life situations, including problems that involve composite shapes

E1.6 solve problems using the relationships between the volume of prisms and pyramids and between the volume of cylinders and cones, involving various units of measure

F. Financial Literacy

Overall expectations

By the end of this course, students will:

F1. Financial Decisions

demonstrate the knowledge and skills needed to make informed financial decisions

Specific expectations

By the end of this course, students will:

Financial Decisions

F1.1 identify a past or current financial situation and explain how it can inform financial decisions, by applying an understanding of the context of the situation and related mathematical knowledge

F1.2 identify financial situations that involve appreciation and depreciation, and use associated graphs to answer related questions

F1.3 compare the effects that different interest rates, lengths of borrowing time, ways in which interest is calculated, and amounts of down payments have on the overall costs associated with purchasing goods or services, using appropriate tools

F1.4 modify budgets displayed in various ways to reflect specific changes in circumstances, and provide a rationale for the modifications

Information for Parents

A parent's guide to Mathematics, Grade 9 (2021)

Resources

Sample course plans for Grade 9 Mathematics

Key Changes – Grade 9 Mathematics, De-streamed (MTH1W), 2021

Ontario Mathematics Curriculum: Grades 7–8–9 Alignment Chart

Introduction to Effective Teaching Practices for the De-streamed Grade 9 Math Classroom

Webinars supporting the math curriculum

Financial literacy modules for students